Example (ii): if $\beta = N^{1/3}$, then $\alpha = 3$, and the computational complexity is $O(N^{4/3})$.

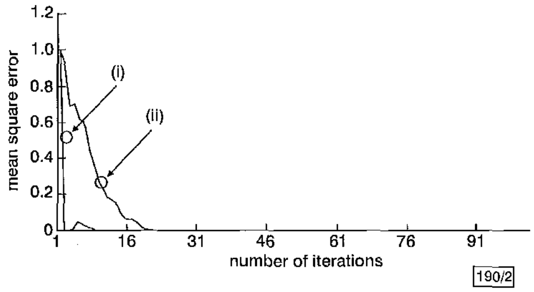

Fig. 2 *Rate of convergence of proposed HRLS and Kalman RLS*
(i) HRLS
(ii) Kalman RLS

Performance evaluation: Simulation results of the proposed scheme are presented in this Section. We assume that the filter has a transversal structure with 16 taps. For the proposed scheme, the 16 taps are organised into a tree of $\alpha = 2$ and $\beta = 4$. A non-minimum phase channel model is used in the simulation. The channel response is $h(k) = \frac{1}{2}[1 + \cos(2\pi(k - 2)/W]$, for k = 1, 2, 3, 4, and $h(k) = 0$, for all other k. An eigenvalue spread of 6.078 corresponding to $W = 2.9$ is chosen. The input to the filter is the sum of the channel output and an independent white Gaussian noise. Fig. 2 shows the results of the simulation runs; the proposed HRLS scheme converges faster than the Kalman RLS, but the computational complexity is significantly lower. The steady-state mean square error of the HRLS and the Kalman RLS are 0.001055 and 0.001292, respectively.

Conclusion: We have proposed a hierarchical scheme to reduce the computational complexity of RLS and increase its convergence rate. The taps are organized into a logical tree. Through local optimisation, which minimises the mean square errors within each group in each level, we are able to reduce the computational complexity. The proposed hierarchical scheme also can be applied to LMS, or even blind algorithms such as the CMA.

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Robust whitening procedure in blind source separation context

A. Belouchrani and A. Cichocki

An efficient algorithm is presented for robust whitening in the presence of temporally uncorrelated additive noise that may be spatially correlated. This whitening is introduced as a preprocessing step in the blind source separation process. The robust whitening consists in the eigenvalue decomposition of a positive definite linear combination of a set a correlation matrices taken at nonzero lags. The coefficients of the linear combination are computed by a finite step global convergence algorithm. Some numerical simulations are provided to illustrate the effectiveness of the solution.

Introduction: Blind source separation consists of recovering independent signals from their instantaneous mixtures without any *a priori* knowledge of these mixtures. Some approaches estimate the source signals by pre-whitening the sensor data followed by a unitary transformation which jointly diagonalise a set of correlation matrices [1 – 3], cumulant matrices [4] or the newly introduced special time frequency distribution matrices [5]. In [1], the whitening matrix is computed from a time-delayed correlation matrix at a nonzero delay. (The idea of the estimation of the whitening matrix from correlation matrices at nonzero delay was first suggested in [2] (see Section III.3 page 436) and worked out in [3].) However, the authors gave no details of how this matrix was chosen. In their simulations, a correlation matrix at a lag close to the zero lag was chosen to ensure the positive definiteness of the matrix. Note that, in this case, we cannot neglect the influence of the noise unless it is perfectly white. Moreover, by choosing a time lag close to zero we need a high sampling frequency rate. In this Letter, we solve this problem by estimating the whitening matrix from an eigenvalue decomposition of a positive definite matrix which is a linear combination of correlation matrices taken at nonzero lags. The coefficients of the linear combination are determined by using the finite step global convergence algorithm proposed in [6].

Data model: In the context of blind source separation, an n-dimensional observation vector $\mathbf{x}(t)$ is assumed to be generated by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the unknown nonsingular mixing matrix, $\mathbf{s}(t)$ is an n-dimensional vector of source signals that are assumed to be mutually uncorrelated and temporally correlated. The vector $\mathbf{n}(t)$ is an additive noise assumed to be zero mean, temporally white and independent from the source signals. Contrary to classical assumptions, no assumption is made on either its distribution or its spatial correlation properties. Hence, its covariance matrix $E[\mathbf{n}(t)\mathbf{n}(t)^T] = \mathbf{R_n}$ can be a full matrix which is generally unknown. Given the above assumptions, the correlation matrices of the observation take the following structure:

$$\mathbf{R_x}(0) = E[\mathbf{x}(t)\mathbf{x}(t)^T] = \mathbf{A}\mathbf{R_s}(0)\mathbf{A}^T + \mathbf{R_n} \tag{2}$$

$$\mathbf{R_x}(i) = E[\mathbf{x}(t)\mathbf{x}(t-i)^T] = \mathbf{A}\mathbf{R_s}(i)\mathbf{A}^T \quad \text{for } i = 1, ..., 0 \tag{3}$$

The problem is how to estimate the mixing matrix $\mathbf{A}$ up to a permutation matrix and a diagonal matrix using only the observed noisy data $\mathbf{x}(t)$.

Solution: The second-order blind identification (SOBI) algorithm which uses a linear combination of correlation matrices to estimate the whitening matrix, is as follows:
(i) Estimate the correlation matrices and compute a singular value decomposition of the $n \times K$ matrix set $R = [\hat{\mathbf{R}}_x(1) \ldots \hat{\mathbf{R}}_x(K)]$

$$R = \mathbf{U}_R \boldsymbol{\Sigma} \mathbf{V}^T \tag{4}$$

where $\mathbf{U}_R \in \mathbb{R}^{n\times n}$ and $\mathbf{V} \in \mathbb{R}^{nK\times nK}$ are orthogonal matrices, and $\boldsymbol{\Sigma}$ has nonzero entries at (i, i) position $(1 \le i \le n)$ and zeros elsewhere.
(ii) For $i = 1, ..., K$ compute

$$\mathbf{F}_i = \mathbf{U}_R^T \hat{\mathbf{R}}_\mathbf{x}(i) \mathbf{U}_R \tag{5}$$

(iii) Choose any initial $\alpha \in \mathbb{R}^n$.
(iv) Compute

$$\mathbf{F} = \sum_{i=1}^{K} \alpha_i \mathbf{F}_i \tag{6}$$

Test: compute a Schur decomposition of $\mathbf{F} \in \mathbb{R}^{n\times n}$. If $\mathbf{F}$ is positive definite, go to step (v), otherwise go to Update.
Update: Choose an eigenvector $\mathbf{u}$ corresponding to the smallest eigenvalue of $\mathbf{F}$ and update α by replacing α with $\alpha + \mathbf{d}$, where

$$\mathbf{d} = \frac{[\mathbf{u}^T\mathbf{F}_1\mathbf{u} \cdots \mathbf{u}^T\mathbf{F}_K\mathbf{u}]^T}{\|[\mathbf{u}^T\mathbf{F}_1\mathbf{u} \cdots \mathbf{u}^T\mathbf{F}_K\mathbf{u}]\|} \tag{7}$$

and go to step (iv). This loop is completed in a finite number step (see [6] for proof).

(v) Compute

$$\mathbf{C} = \sum_{i=1}^{K} \alpha_i \hat{\mathbf{R}}_{\mathbf{x}}(i) \tag{8}$$

and perform an eigenvalue decomposition (EVD) of **C**:

$$\mathbf{C} = \mathbf{U}_c \mathrm{diag}[\lambda_1^2 \cdots \lambda_n^2] \mathbf{U}_c^T \tag{9}$$

where λ_i^2s are the singular values of **C**. A whitening matrix is given by

$$\mathbf{W} = \mathrm{diag}[\lambda_1 \cdots \lambda_n]^{-1} \mathbf{U}_c^T \tag{10}$$

(vi) Form the whitened correlation matrices:

$$\underline{\hat{\mathbf{R}}}_{\mathbf{x}}(i) = \mathbf{W}\hat{\mathbf{R}}_{\mathbf{x}}(i)\mathbf{W}^T \quad \text{for} \quad i = 1, \ldots, K \tag{11}$$

(vii) A unitary matrix **U** is obtained as joint diagonaliser of the set $\{\underline{\hat{\mathbf{R}}}_{\mathbf{x}}(i) | i = 1, \ldots, K\}$.
(vii) The source signals are estimated as

$$\hat{\mathbf{s}}(t) = \mathbf{U}^T \mathbf{W} \mathbf{x}(t) \tag{12}$$

and the mixing matrix is estimated as

$$\hat{\mathbf{A}} = \mathbf{W}^{-1}\mathbf{U} \tag{13}$$

The use of this algorithm for windowed correlation matrices of nonstationary signals as in [1] or for cumulant matrices is straightforward.

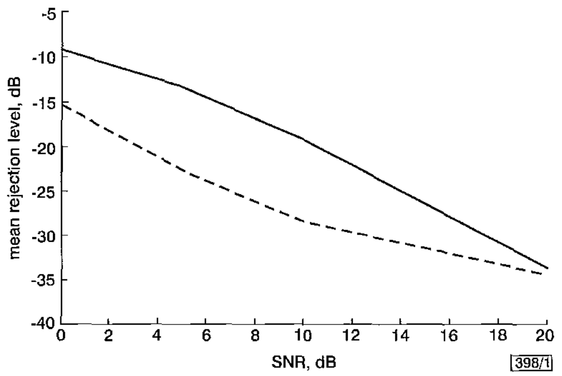

Fig. 1 *Mean rejection level against SNR for coefficient of noise correlation of* $\rho = 0.9$

– – – – whitening using a linear combination of correlation matrices
——— whitening using the autocorrelation matrix

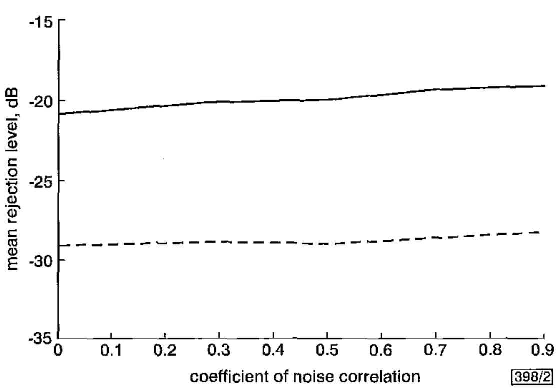

Fig. 2 *Mean rejection level against coefficient of noise correlation* ρ *for SNR = 10dB*

– – – – whitening using a linear combination of correlation matrices
——— whitening using the autocorrelation matrix

Simulation results: The simulated sources are two Gaussian signals located in the frequency domain around the normalised frequency 0.5 and 0.6, respectively. Two sensors are considered. The pth element of the qth column of the mixing matrix **A** is $e^{2i\pi p\theta_q}$ with $\theta_1 = 0.2$ and $\theta_2 = 0, 4$. The additive noise is generated from a zero mean and temporally white Gaussian process with the following covariance matrix:

$$\mathbf{R}_n = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

where σ^2 is the noise power and ρ is the coefficient of noise correlation. We consider here four correlation matrices. Figs. 1 and 2 display the rejection level defined as $[1/n(n-1)]\Sigma_{p\neq q}|(\hat{\mathbf{A}}\#\mathbf{A})_{pq}|^2$ against the signal-to-noise ratio (SNR) (for $\rho = 0.9$) and against the coefficient of the noise correlation (ρ) (for SNR = 10dB), respectively. These plots show the performance of SOBI when the whitening uses correlation matrix (full line) and when it uses a combination of set correlation matrices at a nonzero time lag. The plots show a significant increase in performance (approaching 8dB) when whitening with the combination of a set of correlation matrices at a nonzero time lag.

Conclusion: The blind source separation method presented in [1] and based on the same idea as SOBI [2, 3], estimates a whitening matrix from a time-delayed correlation matrix at a nonzero delay. As this matrix is not guaranteed to be positive definite for some delays, a whitening matrix cannot always be computed from the suggested correlation matrix. To solve this problem, we have proposed using a positive definite linear combination of a set of correlation matrices taken at nonzero time lags to compute the whitening matrix. The coefficients of this linear combination are computed using a finite step global convergence algorithm. The provided simulations show a significant increase in performance when whitening with the combination of a set of correlation matrices at nonzero time lags.

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Minimum cost topology optimisation of ATM networks using genetic algorithms

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The application of a specialised genetic algorithm to the solution of the NP-complete ATM network topology design and capacity assignment problem is considered. It is shown that the developed binary encoding scheme combined with different genetic operator representations and the use of elitism is ideally suited to this type of problem and that computational techniques using this approach lead to a rapid and effective solution for networks of this class.