Robust Blind Source Separation utilizing second and fourth order statistics

Pando Georgiev[1] and Andrzej Cichocki[2]

[1] Brain Science Institute, RIKEN, Wako-shi, Saitama, 351-0198, Japan, On leave from the Sofia University "St. Kl. Ohridski", Bulgaria

[2] Brain Science Institute, RIKEN, Wako-shi, Saitama, 351-0198, Japan, On leave from the Warsaw University of Technology, Poland

{georgiev, cia}@bsp.brain.riken.go.jp

Abstract. We introduce identifiability conditions for the blind source separation (BSS) problem, combining the second and fourth order statistics. We prove that under these conditions, well known methods (like eigen-value decomposition and joint diagonalization) can be applied with probability one, i.e. the set of parameters for which such a method doesn't solve the BSS problem, has a measure zero.

1 Introduction

The interest of blind signal processing, especially, independent component analysis (ICA) has been increased recently, due to its potential applications in many areas, including brain signal processing and other biomedical signal processing, speech enhancement, wireless communication, geophysical data processing, data mining, etc. (see e.g. [11] and references therein).

The problem of blind source separation (BSS) is formulated as follows: we can observe sensor signals $\mathbf{x}(k) = [x_1(k), \ldots, x_n(k)]^T$ which are modeled as

$$\mathbf{x}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where $\mathbf{H}$ is $n \times n$ non-singular unknown mixing matrix, $\mathbf{s}(k) = [s_1(k), \ldots, s_n(k)]^T$ is a vector of unknown zero mean source signals and $\mathbf{n}(k)$ is a vector of additive noise. Assume that $\mathbf{n}$ has independent components (with zero means), which are independent also with $s_i, i = 1, ..., n$.

Our objective is to estimate the mixing matrix $\mathbf{H}$ and/or a separating matrix $\mathbf{W} = \mathbf{H}^{-1}$ and source signals simultaneously.

In this paper we consider an unified model of of source signals and additive noise, which is white of order 2 and 4. We assume that all source signals are uncorrelated of order 2 and 4, as some of the source signals (we don't know which) are white of order 4 but colored of order 2 (for instance colored Gaussian signals) and the rest are white of order 2 and colored of order 4. We introduce a new sufficient conditions for separation (see condition (**DCF**(P)) below) stating that, the sources have different autocorrelation functions or different cumulant functions of fourth order (depending on time delay). These conditions can be considered as a generalization of those ones described in [6] and [14] (for second order statistics) and used in [5].

The use of second statistics approach for blind separation of temporally correlated sources has been developed and analyzed by many researchers, including [1]-[3], [5]- [8], [12],[14], etc.

2 Covariance and cumulant matrices

Define a covariance matrix of the sensor (resp. source) signals by

$$\mathbf{R_x}(p) = E\{\mathbf{x}\mathbf{x}_p^T\}, \text{ (resp. } \mathbf{R_s}(p) = E\{\mathbf{s}\mathbf{s}_p^T\}), \tag{2}$$

where E is the mathematical expectation, $\mathbf{x}_p = \mathbf{x}(k-p), \mathbf{x} = \mathbf{x}(k), \mathbf{s}_p = \mathbf{s}(k-p), \mathbf{s} = \mathbf{s}(k)$and define the symmetric matrix $\tilde{\mathbf{R}}_\mathbf{x}(p) = \frac{1}{2}\big(\tilde{\mathbf{R}}_\mathbf{x}(p) + \tilde{\mathbf{R}}_\mathbf{x}(p)^T\big)$.

Define a fourth order cumulant matrix $\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p}$ of the sensor signals as follows:

$$\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p} = E\{\mathbf{x}\mathbf{x}^T(\mathbf{x}_p^T\mathbf{x}_p)\} - E\{\mathbf{x}\mathbf{x}^T\}\mathrm{tr}E\{\mathbf{x}_p\mathbf{x}_p^T\} - 2E\{\mathbf{x}\mathbf{x}_p^T\}E\{\mathbf{x}_p\mathbf{x}^T\}$$

and define the symmetric matrix $\tilde{\mathbf{C}}^{2,2}_{\mathbf{x},\mathbf{x}_p} = \frac{1}{2}\big(\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p} + (\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p})^T\big)$. It is easy to see that the (i,j)-th element of $\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p}$ is

$$C^{2,2}_{\mathbf{x},\mathbf{x}_p}(i,j) = \sum_{l=1}^{n} \mathbf{cum}\{x_i(k), x_j(k), x_l(k-p), x_l(k-p)\}$$

(see [4] for more general cumulant matrices).

Similarly we define analogous matrices $\mathbf{C}^{2,2}_{\mathbf{s},\mathbf{s}_p}$ and $\tilde{\mathbf{C}}^{2,2}_{\mathbf{s},\mathbf{s}_p}$ for the source signals $\mathbf{s}(k)$. Recall that a signal s is white of order 2 (resp. white of order 4) if

$$E\{s(k)s(k-p)\} = 0, \forall p \geq 1 \text{ (resp. } \mathbf{cum}\{s(k-p_1), s(k-p_2), s(k-p_3), s(k-p_4)\} = 0$$

for every $p_i \geq 1, i = 1, ..., 4$) (see [13]).

In a linear data model (1), if the noise $\mathbf{n}$ is white of order 4, and the mixing matrix H is orthogonal, then the time-delayed cumulant matrices of the observation vector $\mathbf{x}(k)$ for any $p \neq 0$ satisfy $\tilde{\mathbf{C}}^{2,2}_{\mathbf{x},\mathbf{x}_p} = \mathbf{H}\tilde{\mathbf{C}}^{2,2}_{\mathbf{s},\mathbf{s}_p}\mathbf{H}^T$. If $\mathbf{n}$ is white of order 2, then $\tilde{\mathbf{R}}_\mathbf{x}(p) = \mathbf{H}\tilde{\mathbf{R}}_\mathbf{s}(p)\mathbf{H}^T$.

Note that $\mathbf{C}^{2,2}_{\mathbf{x},\mathbf{x}_p}$ (resp. $\mathbf{R_x}(p)$) is symmetric (but not necessary positive definite), if $\mathbf{C}^{2,2}_{\mathbf{s},\mathbf{s}_p}$ (resp. $\mathbf{R_s}(p)$) is a diagonal matrix, but in order to avoid the effect of computational errors (which could destroy the symmetricity), we use $\tilde{\mathbf{C}}^{2,2}_{\mathbf{x},\mathbf{x}_p}$ (resp. $\tilde{\mathbf{R}}_\mathbf{s}(p)$).

3 Robust Orthogonalization

In our method below we need the global mixing matrix to be orthogonal. The standard whitening procedure is not acceptable, since it enhances the noise, especially when the number of sensors is equal to the number of sources and the problem is ill conditioned. We use a preprocessing procedure, which is not sensitive to additive noise. This orthogonalization procedure allows us to define a new orthogonal mixing matrix for the preprocesed data. The idea is to use time-delayed cumulant (resp. covariance) matrices which are not sensitive to additive white noise of order 4 (resp. of order 2) and construct a positive definite matrix from their linear combination (for sufficiently large number of samples). Such a problem for white noise of order 2 is solved in [2] by a finite-step global convergence algorithm [15].

Let $P = \{p_1, ..., p_L\}$ be a set of positive integers with L elements. Denote

$$\mathbf{cum}_{s_i}(p) = \mathbf{cum}\{s_i(k), s_i(k), s_i(k-p), s_i(k-p)\}$$

and assume that the vectors $\left\{\left(\mathbf{cum}_{s_i}(p_1), ..., \mathbf{cum}_{s_i}(p_L)\right)\right\}_{i=1}^{n}$ (resp. vectors $\left(E\{ss_{p_1}\}, ..., E\{ss_{p_L}\}\right)_{i=1}^{n}$) are linearly independent. These conditions are necessary in order to be realized the finite-step global convergence algorithm [15] in Step 1 of the algorithm below.

The robust orthogonalization algorithm can be summarized as follows.

Algorithm Outline: Robust Orthogonalization

1. Find by the finite-step global convergence algorithm [15] a set of parameters $\{\alpha_i\}_{i=1}^{L}$ such that the matrix $\mathbf{C_x}(\boldsymbol{\alpha}) = \sum_{i=1}^{L} \alpha_i \tilde{\mathbf{C}}^{2,2}_{\mathbf{x},\mathbf{x}_{p_i}}$ (resp. $\mathbf{C_x}(\boldsymbol{\alpha}) = \sum_{i=1}^{L} \alpha_i \tilde{\mathbf{R}}_{\mathbf{x}}(p_i)$) is positive definite.
2. Perform an eigenvalue-decomposition of $\mathbf{C_x}(\boldsymbol{\alpha})$, $\mathbf{C_x}(\boldsymbol{\alpha}) = \mathbf{U_x}\boldsymbol{\Lambda}_\mathbf{x}\mathbf{U}_\mathbf{x}^T$, where the entries of diagonal matrix $\boldsymbol{\Lambda}_\mathbf{x}$ are the positive eigenvalues of $\mathbf{C_x}(\boldsymbol{\alpha})$ and compute the preprocessing matrix $\mathbf{Q} = \boldsymbol{\Lambda}_\mathbf{x}^{-\frac{1}{2}}\mathbf{U}_\mathbf{x}^T$.
3. Compute the preprocesed data $\mathbf{z}(k) = \mathbf{Qx}(k) = \mathbf{QHs}(k)$.

Remark 1 By defining a new mixing matrix as $\mathbf{A} = \mathbf{QHD}^{\frac{1}{2}}$, where $\mathbf{D} = \sum_{i=1}^{K} \alpha_i \tilde{\mathbf{C}}^{2,2}_{\mathbf{s},\mathbf{s}_{p_i}}$ (resp. $\mathbf{D} = \sum_{i=1}^{K} \alpha_i \tilde{\mathbf{R}}_{\mathbf{s}}(p_i)$) is a diagonal (scaling) matrix with positive entries, we see that $\mathbf{C_z}(\boldsymbol{\alpha}) = \mathbf{AA}^T = \mathbf{I}_n((n \times n)$ unit matrix), so, the matrix $\mathbf{A}$ is orthogonal. This orthogonality condition is necessary for performing separation of signals using either symmetric EVD, or joint diagonalization. It should be noted that in contrast to the standard prewhithening procedure for our robust orthogonalization generally $E\{\mathbf{zz}^T\} \neq \mathbf{I}_m$, but we have $\sum_{i=1}^{K} \alpha_i \tilde{\mathbf{C}}^{2,2}_{\tilde{\mathbf{s}},\tilde{\mathbf{s}}_{p_i}} = \mathbf{I}_n$ and $\sum_{i=1}^{K} \alpha_i \tilde{\mathbf{C}}^{2,2}_{\mathbf{z},\mathbf{z}_{p_i}} = \mathbf{I}_n$. So, our model is $\mathbf{z} = \mathbf{A}\tilde{\mathbf{s}} + \mathbf{Qn}$, where $\tilde{\mathbf{s}} = \mathbf{D}^{-\frac{1}{2}}\mathbf{s}$.

4 Sufficient Conditions for Simultaneous Blind Source Separation

We introduce the following conditions, called (**DCF**(P)) (different cumulant functions):

$$\forall i, j \neq i\ \exists l_{i,j} \in \{1, ..., L\}: \quad \text{either } E\{s_i(t)s_i(t - p_{l_{i,j}})\} \neq E\{s_j(t)s_j(t - p_{l_{i,j}})\}$$
$$\text{or } \mathbf{cum}_{s_i}(p_{l_{i,j}}) \neq \mathbf{cum}_{s_j}(p_{l_{i,j}}),$$

i.e. the sources have different autocorrelation or cumulant functions of fourth order on the set P.

Define the following matrices for the chosen set P of time delays for $\mathbf{z}$ and $\mathbf{s}$ respectively, where $\mathbf{b}, \mathbf{c} \in \mathbb{R}^L$:

$$\mathbf{Z}(\mathbf{b}, \mathbf{c}) = \sum_{i=1}^{L} \left(b_i \tilde{\mathbf{R}}_{\mathbf{z}}(p_i) + c_i \tilde{\mathbf{C}}^{2,2}_{\mathbf{z},\mathbf{z}_{p_i}} \right), \quad \mathbf{S}(\mathbf{b}, \mathbf{c}) = \sum_{i=1}^{L} \left(b_i \tilde{\mathbf{R}}_{\mathbf{s}}(p_i) + c_i \tilde{\mathbf{C}}^{2,2}_{\mathbf{s},\mathbf{s}_{p_i}} \right). (3)$$

We recall that the source signals are *uncorrelated*, if $\mathbf{R}_{\mathbf{s}}(p)$ are diagonal matrices for every $p \geq 1$. If the source signals are statistically independent, then this condition is satisfied, but the converse assertion is not always true. Note that the diagonal elements of $\mathbf{R}_{\mathbf{s}}(p)$ are $E\{\mathbf{s}_i(k)\mathbf{s}_i(k-p)\}$. We say that the source signals are *colored*, if for some $p_0 \geq 1$ the matrix $\mathbf{R}_{\mathbf{s}}(p_0)$ has a nonzero diagonal element. We shall say that the source signals are *uncorrelated of order* 4, if $\mathbf{C}^{2,2}_{\mathbf{s},\mathbf{s}_p}$ are diagonal matrices for every $p \geq 1$ with diagonal elements $\mathbf{cum}_{s_i}(p)$. If the source signals are statistically independent, then this condition is satisfied, but the converse assertion is not always true. We shall say that the sources are *colored of order* 4, if for some $p_0 \geq 1$, $\mathbf{cum}_{s_i}(p_0)$ is nonzero. So, if $s_i, i = 1, ..., n$ are uncorrelated of order 4 and colored of order 4, then for some $p_0 \geq 1$, the matrix $\mathbf{C}^{2,2}_{\mathbf{s},\mathbf{s}_{p_0}}$ is a nonzero diagonal matrix.

Theorem 1. *Assume that the mixing matrix* $\mathbf{A}$ *is orthogonal, the source signals are uncorrelated of order* 2 *and* 4*, condition (*$\mathbf{DCF}$*(P)) is satisfied and the additive noise* $\mathbf{n}$ *is white of order 2 and* 4*. Then:*

(a) the matrix $\mathbf{Z}(\mathbf{b}, \mathbf{c})$ *is symmetrical and can be decomposed as* $\mathbf{Z}(\mathbf{b}, \mathbf{c}) = \mathbf{A}\mathbf{S}(\mathbf{b}, \mathbf{c})\mathbf{A}^T$*;*

(b) there exist vectors $\mathbf{b}, \mathbf{c} \in \mathbb{R}^L$ *such that the matrix* $\mathbf{Z}(\mathbf{b}, \mathbf{c})$ *has distinct eigenvalues. Furthermore, the set* $B(L)$ *of all vectors* $(\mathbf{b}, \mathbf{c}) \in \mathbb{R}^{2L}$ *with this property form an open subset of* $\mathbb{R}^{2L}$*, whose complement has a measure zero;*

(b) if $\mathbf{U}$ *is given from an EVD of the matrix* $\mathbf{Z}(\mathbf{b}, \mathbf{c})$ *for some* $\mathbf{b}, \mathbf{c} \in B(L)$*, i.e.* $\mathbf{Z}(\mathbf{b}, \mathbf{c}) = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^T$*, then the estimating mixing matrix is* $\hat{\mathbf{A}} = \mathbf{U}$ *and the separating matrix is* $\mathbf{W} = \hat{\mathbf{A}}^T = \mathbf{U}^T$ *(up to multiplication with arbitrary permutation and diagonal nonsingular scaling matrices).*

Proof. (a) The assertion follows from the properties of the cumulants.

(b) Since $s_i, i = 1, ..., n$ are uncorrelated, $\mathbf{S}(\mathbf{b}, \mathbf{c})$ is a diagonal matrix and by Lemma 1, $\mathbf{Z}(\mathbf{b}, \mathbf{c}) = \mathbf{A}\mathbf{S}(\mathbf{b}, \mathbf{c})\mathbf{A}^T$. Observe that the matrices $\mathbf{Z}(\mathbf{b}, \mathbf{c})$ and $\mathbf{S}(\mathbf{b}, \mathbf{c})$ have the same eigenvalues. It is easy to see that the complement of $B(L)$ is a finite union of subspaces of $\mathbb{R}^{2L}$. If we prove that $B(L)$ is nonempty, then every of these subspaces must be proper (i.e. different from $\mathbb{R}^{2L}$), consequently, with a measure zero (with respect to $\mathbb{R}^{2L}$), therefore the complement of $B(L)$ must have a measure zero too.

Choose $\mathbf{b}, \mathbf{c} \in \mathbb{R}^{2L}$ arbitrary. Let $\{\sigma_i(\mathbf{b}, \mathbf{c})\}_{i=1}^n$ be the diagonal elements of the matrix $\mathbf{S}(\mathbf{b}, \mathbf{c})$. Assume that two diagonal elements of the matrix $\mathbf{S}(\mathbf{b}, \mathbf{c})$ are equal, for example $\sigma_1(\mathbf{b}, \mathbf{c}) = \sigma_2(\mathbf{b}, \mathbf{c})$. Let $l_{1,2}$ be the index defined by condition ($\mathbf{DCF}$(P)). If $E\{s_1(t)s_1(t - p_{l_{1,2}})\} \neq E\{s_2(t)s_2(t - p_{l_{1,2}})$ then choose a vector $\mathbf{b}(1,2)$, which is different from $\mathbf{b}$ only in the component $b_{l_{1,2}}$. Otherwise, by condition $\mathbf{DCF}(\mathrm{P})$, we must have $\mathbf{cum}_{s_i}(p_{l_{i,j}}) \neq \mathbf{cum}_{s_j}(p_{l_{i,j}})$ and then we choose a vector $\mathbf{c}(1,2)$, which is different from $\mathbf{c}$ only in the component $c_{l_{1,2}}$, and put $\mathbf{b}(1,2) = \mathbf{b}$. Then $\sigma_1(\mathbf{b}(1,2), \mathbf{c}(1,2)) \neq \sigma_2(\mathbf{b}(1,2), \mathbf{c}(1,2))$, because of the condition ($\mathbf{DCF}(\mathrm{P})$). If all diagonal elements of $\mathbf{S}(\mathbf{b}(1,2), \mathbf{c}(1,2))$ are different, we finish the proof. If not, suppose that $\sigma_i(\mathbf{b}(1,2), \mathbf{c}(1,2)) = \sigma_j(\mathbf{b}(1,2), \mathbf{c}(1,2))$ for some indexes i and j. We can change a little either the component $b_{l_{i,j}}$ of the vector $\mathbf{b}(1,2)$ or $c_{l_{i,j}}$ of the vector $\mathbf{c}(1,2)$ (keeping the other components the same) such that for the new vector $(\mathbf{b}(i,j), \mathbf{c}(i,j))$ to be satisfied $\sigma_i(\mathbf{b}(i,j), \mathbf{c}(i,j)) \neq \sigma_j(\mathbf{b}(i,j), \mathbf{c}(i,j))$ (because of condition ($\mathbf{DCF}(\mathrm{P})$) and since $\sigma_1(\mathbf{b}(i,j), \mathbf{c}(i,j)) \neq \sigma_2(\mathbf{b}(i,j), \mathbf{c}(i,j))$. Continuing in such a way, for any couple $(k,r), k \neq r$ for which $\sigma_k(\mathbf{b}(k',r'), \mathbf{c}(k',r')) = \sigma_r(\mathbf{b}(k',r'), \mathbf{c}(k',r'))$ (where $(\mathbf{b}(k',r'), \mathbf{c}(k',r'))$ is the vector considered in the previous step), we make small change either of $b_{l_{k,r}}$ or of $c_{l_{k,r}}$ keeping the pair-wise difference of the diagonal elements considered in the previous steps and obtain a vector $(\mathbf{b}(k,r), \mathbf{c}(k,r))$ for which $\sigma_k(\mathbf{b}(k,r), \mathbf{c}(k,r)) \neq \sigma_r(\mathbf{b}(k,r), \mathbf{c}(k,r))$. So, after finite number of steps we obtain a vector $(\mathbf{b}^*, \mathbf{c}^*)$ for which the diagonal elements of $\mathbf{S}(\mathbf{b}^*, \mathbf{c}^*)$ are distinct. This proves the non-emptiness of the set $B(L)$ and finishes the proof of (b).

(c) This follows from the well known facts of linear algebra [10]. ∎

Corollary 1. *Under assumption (ii) of Theorem 1, an estimation of the mixing matrix is possible from the EVD of the cumulant matrix $\mathbf{C}^{2,2}_{\mathbf{z},\mathbf{z}_p}$, if the sources has different cumulants of fourth order for a fixed time delay p, i.e., if*

$$\mathbf{cum}(s_i(k), s_i(k), s_i(k-p), s_i(k-p)) \neq \mathbf{cum}(s_j(k), s_j(k), s_j(k-p), s_j(k-p))$$

for every $i \neq j$. When $p = 0$, the above condition means that the source signals have different kurtosis; in this case the conclusion is also true, if in addition, the noise is Gaussian.

Remark 1. When the mixing matrix is not orthogonal, the condition for separation can be obtained having in mind the orthogonalization procedure: so, condition $\mathbf{DCF}(\mathrm{P})$ should be satisfied for the signals $\tilde{\mathbf{s}} = \mathbf{D}^{-\frac{1}{2}}\mathbf{s}$ (see Remark 1). In case of usual pre-whitening, the matrix $\mathbf{D}$ is equal to $E\{\mathbf{s}\mathbf{s}^T\}$ and, when all source signals are colored of order 2, we recover identifiability conditions presented in [14], Theorem 2.

Remark 2. Theorem 1 gives another proof of the mathematical foundation of the SOBI algorithm [3]. We shall present simulation examples at the conference.

5 Conclusion

We develop an unified approach by second and high order statistics to BSS problem and presented new identifiability conditions. This approach gives justification of EVD and joint diagonalization procedures for solving BSS problem. It has some advantages, among which robustness to additive noise and possibility to implement high speed algorithms for symmetric eigenvalue problems [9] and joint diagonalization.

References

1. S. Amari "ICA of temporally correlated signals - learning algorithm". *Proc. ICA '99: International workshop on blind signal separation and independent component analysis*, Aussois, France, pp. 13-18, Jan. 1999.
2. A. Belouchrani and A. Cichocki A. "Robust whitening procedure in blind separation context". *Electronics Letters*, Vol. 36 No.24, pp. 2050-2051, 2000.
3. A. Belouchrani, K. A. Meraim, J.-P. Cardoso and E. Moulines "A blind source separation technique using second order statistics." *IEEE Trans. on Signal Processing*, Vol. 45, no. 2, pp. 434-44, Feb. 1997
4. J.-F. Cardoso, "High-order contrasts for independent component analysis" Neural Computation, vol. 11, no 1, pp. 157–192, Jan. 1999.
5. C. Chang, Z. Ding, S. F. Yau and F. H. Y. Chan "A matrix-pencil approach to blind separation of colored nonstationary signals". *IEEE Trans. Signal Proc.*, Vol. 48, No. 3, pp. 900-907, Mar. 2000.
6. C. Chang, S.F. Yau, P. Kwok, F.H.Y. Chan, F.K. Lam, *Uncorrelated and component analysis for blind source separation*, Circuits Systems and Signal Processsing, Vol. 18, No.3, pp. 225-239, 1999.
7. A. Cichocki, and R. Thawonmas " On-line algorithm for blind signal extraction of arbitrarily distributed, but temporally correlated sources using second order statistics". *Neural Processing Letters* Vol. 12, pp. 91-98, Aug. 2000.
8. S. Choi and A. Cichocki "Blind separation of nonstationary and temporally correlated sources from noisy mixtures". *IEEE Workshop on Neural Networks for Signal Processing, NNSP'2000*, Sydney, Australia, pp. 405-414, Dec. 2000.
9. A. Edelman, T. A. Arias and A. T. Smith "The geometry of algorithms with orthogonality constraints". *SIAM J. Matrix Anal. Appl.*, Vo. 20, No.2, pp. 303-353, 1998.
10. G. H. Golub and C. F. Van Loan *Matrix Computation.* J. Hopkins Univ. Press, 1989.
11. A. Hyvarinen, J. Karhunen and E. Oja, "Independent Component Analysis", John Wiley & Sons, 2001.
12. L. Molgedey and H. G. Schuster "Separation of a mixture of independent signals using time-delayed correlations. *Physical Review Letters*, Vol. 72, No. 23, pp. 3634-3637, 1994.
13. C. L. Nikias and A. Petropulu "Higher-order spectra analysis. A nonlinear signal processing framework", Prentice Hall Signal Processing Series, 1993.
14. L. Tong, R. Liu, V.C. Soon and Y.F. Huang, *Indeterminacy and Identifiability of Blind Identification*, IEEE Transactions on Circuits and Systems, Vol. 38, No.5, pp. 499-509, May 1991.
15. L. Tong, Y. Inouye and R. Liu " A finite-step global algorithm for the parameter estimation of multichannel MA processes". *IEEE Trans. Signal Proc.*, 40(10): 2547-2558, 1992.