Indeterminacy and Identifiability of Blind Identification

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Abstract—**Blind identification of source signals is studied from both theoretical and algorithmic aspects. A mathematical structure is formulated from which the acceptable indeterminacy is represented by an equivalence relation. The concept of identifiability is then defined. Two identifiable cases are shown along with blind identification algorithms.**

I. Introduction

BLIND identification is an emerging field of fundamental research with a wide range of applications. It has been motivated by practical problems that involve multiple source signals and multiple sensors, and which share a common objective, i.e., separating and estimating the sources signals *without knowing the characteristic of the transmission channel*. The problem of blind identification can be depicted by the block diagram of Fig. 1.

1.1. Applications of Blind Identification

In array signal processing (see, e.g., [1], [2]), the sensor array receives signals from multiple sources as shown in Fig. 2. The source signals may be totally unknown as in the case of passive sonar applications. Furthermore, the transmission channel, i.e., the ocean environment, is also unknown and time varying.

In medical science, it is of great interest to determine the firing patterns of the neuronal signals from electromyograms (EMG) [3]. These EMG signals, for noninvasive purpose, are usually measured from electrodes at the skin level. The characteristic of the medium between the point where a neuronal signal is initiated and the electrodes at the skin level is unknown, and it varies from person to person.

In designing voice-controlled machines, the machine has to be able to recognize the commanding voice in a noisy environment, which may consist of other voices as well as noise [4]. The characteristics of the medium are unknown because, among others, the relative positions of the multiple sources and multiple receivers are not known *a priori*. A similar situation exists when one needs to communicate accurately in an aircraft cockpit environment [5].

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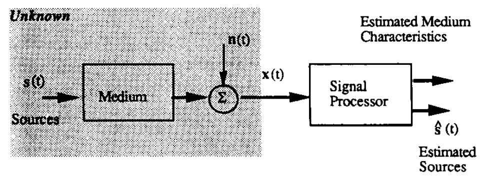

Fig. 1. A schematic diagram of blind identification.

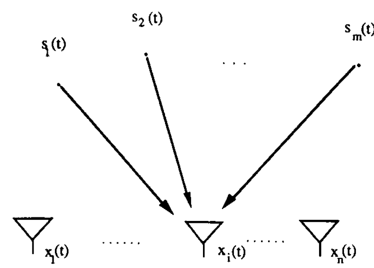

Fig. 2. A typical multiple sources and sensor array problem.

In the area of image reconstruction and restoration ([6] and references therein), the basic problem is to reconstruct the original objects from an image with certain degradation. The degradation caused by various reasons such as the motion of the camera, distortion of the lenses and transmission channel, etc., is unknown.

The applications related to blind identification extend far beyond the field of signal processing.

In semiconductor manufacturing [7], one critical task is to determine the status of some key process parameters (e.g., diffusion times and temperatures, gas fluxes, etc.) from the process testing data (e.g., threshold voltage, drive current, sheet resistance, parasitic capacitance, etc.). In practice, the relation between those process parameters and the test data is unknown.

In circuit testing and diagnosis, one may want to determine the randomly located input signals from the output signals. The input signals could be thermal noise, or compensate signals of a faulty element [8]. A solution to

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the blind identification problem can provide the information of the locations as well as the nature of these signals. Note that the transfer function cannot be determined *a priori* because the locations of these input signals are unknown.

Factor analysis [9, chs. 6 and 7], [10, ch. 14], [11], a widely used approach in the areas of behavioral and health science, can also be formulated as a blind identification problem in which independent "factors" are extracted from observations without knowing the relationship between the observations and the more fundamental quantities—the factors and the separation.

In summary, the commonality of the above examples is that the identification and the separation of multiple source signals need to be achieved, *without knowing the characteristics of the channel*. In the current literature, blind identification is treated individually for each specific application so that special properties of the underlying application can be incorporated. It is important, however, to find the theoretical limitations of the common solutions to the aforementioned blind identification problems.

1.2. *Main Objective and Organization of the Paper*

The first objective of this paper is to develop a general mathematical structure for the blind identification problems. With this structure, the concept of identifiability can be defined and studied. Furthermore, it provides the theoretical support for the blind identification algorithms presented later.

This paper is organized as follows. In Section II, the problem statements are given along with a literature review. In Section III, a mathematical framework is developed based on the concepts of identification space, waveform-preserving equivalence, and identifiability. Theoretical issues related to identifiability are investigated. Two identifiable cases are presented in Section IV. In Section V, two algorithms for blind identification problems—EFOBI and AMUSE—are presented and their performance evaluation and an illustrative example are presented in Section VI. Finally, concluding remarks are given in Section VII.

II. Problem Statements and Literature Review

The blind identification of a linear memoryless channel is our main concern in this paper, not only because such a model is mathematically tractable, but also because this model is relatively accurate for many applications, including semiconductor manufacturing process [7], factor analysis [9], narrow-band array signal processing [13], and image reconstruction [6]. In addition, a better understanding of the blind identification problems for the memoryless case would certainly provide a thrust for the investigation of the general models.

Consider the following identification problem:

$$x(t) = As(t) + n(t), \qquad t = 1, 2, \cdots \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the observation vector, $s(t) \in \mathbb{R}^m$ is the vector of unknown source signals, $n(t) \in \mathbb{R}^n$ is the additive random noise vector, and $A \in \mathbb{R}^{n \times m}$ is the parameter matrix that characterizes the medium or the channel. The *blind identification* is to identify *both A and* $s(\cdot)$ from $x(\cdot)$. It should be noted that the blind identification problem differs from the conventional system identification problem in the assumption of source signals. For the latter case, it usually assumes that either $s(\cdot)$ is known or it is a white noise. In blind identification, such an assumption is removed.

One objective of this paper is to study the indeterminacy and identifiability of the problem of blind identification as well as the blind identification algorithms. Throughout this paper, A_0 and $s_0(\cdot)$ stand for the actual channel parameter matrix and the actual source signals, respectively. In addition, we impose the following assumptions on the model equation (1).

Basic Model Assumption:

(A1) $A_0 \in \mathbb{R}^{n \times m}$ has full column rank, i.e.,

$$\text{rank}(A_0) = m.$$

(A2) $s_0(\cdot)$ is a zero-mean stationary process with a nonsingular covariance matrix $R_{s_0} \equiv E\{s_0(t)s_0^t(t)\}$.

(A3) $n(\cdot)$ is a zero-mean wide sense stationary (WSS) white Gaussian noise process.

(A4) Source signal $s_0(t)$ and noise $n(t)$ are statistically mutually independent.

Blind identification problems have long been studied in the context of factor analysis by statisticians, economists, and psychologists [9]–[11]. One typical approach assumes certain structural condition, such as the position of zero elements, on the matrix A. Such structural conditions are derived for specific applications, and have little general implication.

Eigenstructure-based algorithms have been proposed for identification problems where certain structural information of matrix A is available. The celebrated algorithm MUSIC [13] assumes that the matrix A has a certain structure. Hence its application is primarily in the areas of array signal processing and spectrum estimation. ESPRIT [14], on the other hand, can be applied to a more general class of blind identification problems. The key assumption of ESPRIT is that the parameter matrix A has a special invariant property, namely, $A = \begin{bmatrix} B \\ BD \end{bmatrix}$ where B is arbitrary and D is diagonal. Such an invariant property is guaranteed in certain applications of array signal processing.

Recently, there have been considerable research interests in the general case of blind identification problems on which no particular structural assumptions on A are imposed [15]–[19]. Herault–Jutten [15] and Cardoso [16] studied the cases where the source signals can be assumed to be mutually independent. Unfortunately, the adaptive algorithm proposed by Herault–Jutten and the fourth-order blind identification (FOBI) algorithm by

Cardoso have some critical shortcomings. Herault–Jutten's approach does not guarantee convergence of its estimates. In addition, neither Herault–Jutten's approach nor Cardoso's FOBI algorithm considered the noise effects. The cases when noise cannot be neglected are further studied in [17]–[19]. Although various algorithms have been proposed, the fundamental problem of blind identification of source signals was not well understood.

III. Identification Indeterminacy, Waveform-Preserving Equivalence, and Identifiability

3.1. Identification Space and its Waveform-Preserving Equivalence Classes

In order to formulate the blind identification problem in a proper mathematical framework, we first introduce the concept of identification space which, intuitively speaking, is the collection of source signals and channel parameter matrices that would produce the same observation $x(\cdot)$ as that produced by the actual source signal $s_0(\cdot)$ and the actual channel parameter matrix A_0.

Definition 1: Let $\mathbb{M}_0$ denote the set of (A,s) that satisfies the basic model assumption. Given (x,n), the *identification space* $\mathbb{I}_{(x,n)}$ is defined by

$$\mathbb{I}_{(x,n)} = \{(A,s) \in \mathbb{M}_0 | x(\cdot) = As(\cdot) + n(\cdot)\}.$$

In other words, $(A, s(\cdot)) \in \mathbb{I}_{(x,n)}$ if and only if it satisfies the model assumption and the condition $As(\cdot) = A_0 s_0(\cdot)$.[1]

It is obvious that if $(A_0, s_0) \in \mathbb{I}_{(x,n)}$, then

$$(A_0 M, M^{-1} s_0) \in \mathbb{I}_{(x,n)}$$

for any nonsingular matrix $M \in \mathbb{R}^{m \times m}$. Since every element in the identification space is a legitimate estimate, there is an inherited indeterminacy in the underlying blind identification problem. This indeterminacy is characterized, or parameterized, by an arbitrary nonsingular matrix M. On the other hand, not every $(A,s) \in \mathbb{I}_{(x,n)}$ is a good estimate of (A_0, s_0) in practical applications because neither the waveforms of $s(\cdot)$ nor the statistical properties of $s(\cdot)$ may resemble that of the actual source signal $s_0(\cdot)$. Therefore, it is desirable to determine the class of M that is acceptable for practical applications. The set of acceptable indeterminacies to be presented is motivated by the following observations.

i) For many applications, most relevant information of the source signals is contained in the waveforms of the source signals rather than in the magnitudes of the source signals. Hence the indeterminacy associated with the magnitude of the source signals would be acceptable. In particular, an estimate $s(\cdot)$ is acceptable if $s(\cdot) = \Lambda s_0(\cdot)$ where Λ is any nonsingular *diagonal* matrix.

ii) For blind identification problems, the indeterminacy associated with the order in which the source signals are arranged is acceptable. In other words, an estimate $s(\cdot)$ is acceptable if $s(\cdot) = P s_0(\cdot)$, where P is any *permutation* matrix.

[1] Strictly speaking, the equality should be read as equal almost surely.

The above observations can be translated, in a mathematical language, to a *waveform-preserving* relation between pairs of elements in the identification space.

Definition 2: Two ordered pairs of doublets (A,s) and (A',s') are said to be related by a *waveform-preserving relation* $\Re$, i.e., $(A,s)\Re(A',s')$, if

$$A' = A\Lambda^{-1}P^t \tag{2}$$

$$s'(\cdot) = P\Lambda s(\cdot) \tag{3}$$

for some permutation matrix P, and some nonsingular diagonal matrix Λ.

Note that if two doublets (A,s) and (A',s') are related by $\Re$, then the two source signal vectors differ only by a permutation transformation and/or their respective components by a scalar multiplier. Hence the waveform of the signal is retained under $\Re$. Similarly, the column vectors of two related parameter matrices differ only by a permutation and/or in their norm. The direction of related column vectors is preserved. We shall show that this relation is an equivalence relation.

Proposition 1 (Equivalence Relation): The waveform-preserving relation $\Re$ is an equivalence relation.

Proof: The reflexivity, symmetry, and transivity can be verified directly from Definition 2. □

For notational convenience, $(A,s)\Re(A',s')$ is denoted as $(A,s) \sim (A',s')$ in the sequel. We also use the notation $A \sim A'$ and $s \sim s'$ when there is no confusion. Here, the symbol " $\sim$ " denotes equivalence relation.

Note that the waveforms of the signals are preserved under $\Re$. It is interesting to see that some statistical properties are also preserved, as shown in the following proposition.

Proposition 2 (Statistical Properties under $\Re$): Let $s'(\cdot) = [s_1'(\cdot), s_2'(\cdot), \cdots, s_m'(\cdot)]^t$ and $s(\cdot) = [s_1(\cdot), s_2(\cdot), \cdots, s_m(\cdot)]^t$. If $(A,s) \sim (A',s')$, then we have the following:

i) $\{s_i(t),\ i = 1,2,\cdots,m\}$ are mutually independent iff $\{s_i'(\cdot),\ i = 1,2,\cdots,m\}$ are mutually independent;

ii) denote $R_s(\tau) = E(s(t)s(t-\tau)^t)$ and $R_s'(\tau) = E(s'(t)s'(t-\tau)^t)$. Then $R_s(\tau)$ is diagonal iff $R_s'(\tau)$ is diagonal.

Proof: i) Since $s \sim s'$, we have $s'(t) = P\Lambda s(t)$ for some permutation matrix P and nonsingular diagonal matrix Λ. Consequently, $s_i(t) = \lambda_i s_{k_i}'(t)$ for $i = 1,2,\cdots,m$, where $\{k_1, k_2, \cdots, k_m\}$ is a permutation of $\{1,2,\cdots,m\}$ and $\lambda_i \neq 0$. Hence $\{s_{k_i}',\ i = 1,2,\cdots,m\}$ is a set of mutually independent random variables if and only if $\{s_i,\ i = 1,2,\cdots,m\}$ is a set of mutually independent random variables.

ii) From $s'(t) = P\Lambda s(t)$, we have

$$R_s'(\tau) \equiv E\{s'(t)s'(t-\tau)^t\} = P\Lambda R_s(\tau)\Lambda^t P^t$$

where $R_s(\tau) \equiv E\{s(t)s^t(t-\tau)\}$. If $R_s(\tau)$ is diagonal, then there exists a diagonal matrix Λ' such that

$$P\Lambda R_s(\tau)\Lambda' = \Lambda' P.$$

Hence $R_s'(\tau) = \Lambda' P P^t = \Lambda'$ is diagonal. The converse can be proved similarly. □

3.2. Identifiability

The significance of the equivalence relation $\Re$ on the identification space $\mathbb{I}_{(x,n)}$ is that the unique set of equivalence classes induced by $\Re$ forms a partition. Furthermore, the actual (A_0, s_0) belongs to one and only one equivalence class. The objective of blind identification is therefore to identify the *equivalence class* that contains (A_0, s_0). As was pointed out in the beginning of this section, the inherited indeterminacy makes the identification of actual (A_0, s_0) impossible. In contrast, as we shall show later, the identification of the equivalence class can be made possible.

With the equivalence classes induced by $\Re$, we now ask the following question: *given certain properties satisfied by* (A_0, s_0) *and its equivalent members in* $\mathbb{I}_{(x,n)}$, *how many equivalence classes in* $\mathbb{I}_{(x,n)}$ *also satisfy the same properties*? If there is only one such equivalence class in $\mathbb{I}_{(x,n)}$, then (A_0, s_0) must belong to that equivalence class, hence the equivalence class is *identifiable*. If there is more than one such equivalence class in $\mathbb{I}_{(x,n)}$, then the identification is not unique. We now state the definition of identifiability.

Definition 3 (Identifiability): Let $\mathbb{M}$ be a set of (A, s) that satisfies a model structure. Then $(A_0, s_0) \in \mathbb{I}_{(x,n)}$ is *identifiable with respect to a model* $\mathbb{M}$ if for every $(A, s) \in \mathbb{I}_{(x,n)} \cap \mathbb{M}$, $(A_0, s_0) \sim (A, s)$.

It is important to note that the concept of identifiability is defined with respect to a certain model structure. The specification of a model structure involves specifying the channel properties as well as the signal properties. For example, suppose that the model equation (1) under the basic model assumption is chosen as the model structure. One can analyze whether (A_0, s_0) is identifiable under this model structure. The answer, unfortunately, is negative. For any nonsingular matrix M, $(A_0 M^{-1}, M s_0) \in \mathbb{I}_{(x,n)}$ and satisfies the model structure. However, it is not true that $(A_0 M^{-1}, M s_0) \sim (A_0, s_0)$ for all M. Hence (A_0, s_0) is not identifiable with respect to this model structure. In Section IV, we shall present two model structures for which (A_0, s_0) can indeed be identified.

3.3. Orthogonalization and Identifiability

Here we study transformations between the identification spaces and how the transformation affects the identifiability. In particular, we are interested in the transformations that orthogonalize the column vectors of the channel parameter matrix. Such transforms reduce the complexity of the blind identification problem and play an important role in the algorithm implementations.

Definition 4 (Orthogonal Equivalence Class): An equivalence class defined on $\mathbb{I}_{(x,n)}$ is an *orthogonal equivalence class* if all members in the equivalence class have parameter matrices whose column vectors are orthogonal.

In the following, we shall show that for every $(A, s) \in \mathbb{I}_{(x,n)}$, there exists a matrix $T \in \mathbb{R}^{m \times n}$ such that (TA, s) belongs to an orthogonal equivalence class in $\mathbb{I}_{(Tx, Tn)}$.

By the singular value decomposition (SVD) theorem [20], any matrix $A \in \mathbb{R}^{n \times m}$ has the decomposition

$$A = U_s \Sigma V \tag{4}$$

where $U_s \in \mathbb{R}^{(n \times m)}$ is such that $U_s^t U_s = I$, $V \in \mathbb{R}^{(m \times m)}$ is orthogonal, and Σ is positive definite and diagonal. Now define

$$T \equiv \Sigma^{-1} U_s^t. \tag{5}$$

We then have

$$\begin{aligned} TA &= \Sigma^{-1} U_s^t U_s \Sigma V \\ &= V \end{aligned}$$

which is orthogonal. Now, defining

$$y \equiv Tx \tag{6a}$$

$$B \equiv TA \tag{6b}$$

$$w \equiv Tn \tag{6c}$$

we have

$$y(t) = Bs(t) + w(t). \tag{7}$$

Comparing this equation with (1), it is easy to show that (B, s) satisfies the basic model assumption if and only if (A, s) satisfies the basic model assumption. The difference, though, is that the column vectors of $B = V$ are orthogonal, while those of A are not. It will be shown later that this transformation reduces the complexity of the blind identification problem.

One might have noticed that the matrix T defined by (5) comes directly from the parameter matrix A, which is unknown. Fortunately, the matrix T can be obtained from the statistics of the observation $x(t)$, which is readily available. The construction of T, or its equivalent from the output statistics will be elaborated in Section V.

The transformation (6) induces a mapping f from $\mathbb{I}_{(x,n)}$ to $\mathbb{I}_{(y,w)}$, where $\mathbb{I}_{(y,w)}$ is the identification space defined by model equation (7). In particular, f performs the following operation:

$$f: \mathbb{I}_{(x,n)} \to \mathbb{I}_{(y,w)} \tag{8a}$$

$$(A, s) \mapsto (TA, s). \tag{8b}$$

Instead of identifying (A_0, s_0) from $x(\cdot)$, we can first identify (B_0, s_0) from $y(\cdot)$, which is simpler because the column vectors of B_0 are orthogonal.

Before we endorse such an approach, we need to make some justifications for the transformation f. Particularly, the equivalence classes induced by $\Re$ must be preserved by f. In other words, the image of two equivalent elements in $\mathbb{I}_{(x,n)}$ must be equivalent in $\mathbb{I}_{(y,w)}$.

Proposition 3: The transformation f preserves the equivalence relation $\Re$.

Proof: We need to show that if $(A,s) \sim (A',s')$, then $f((A,s)) \sim f((A',s'))$. Suppose $(A,s) \sim (A',s')$. Then for some permutation matrix P and nonsingular diagonal matrix Λ,

$$A' = A\Lambda^{-1}P^t$$
$$s'(t) = P\Lambda s(t).$$

With f defined in (8), we have

$$f((A',s')) = (TA',s') = (TA\Lambda^{-1}P^t, P\Lambda s) \sim (TA,s) = f((A,s)). \quad \square$$

An important implication of Proposition 3 is that the identifiability of $(A_0,s_0) \in \mathbb{I}_{(x,n)}$ is preserved by f, i.e., if $(A_0,s_0) \in \mathbb{I}_{(x,n)}$ is identifiable with respect to some model assumption $\mathbb{M}$ on the source signals, then $(B_0,s_0) \in \mathbb{I}_{(y,w)}$ is also identifiable with respect to $\mathbb{M}$. It is equally important to investigate the converse statement. This is nontrivial because f is not injective whenever $m < n$.

Theorem 1 (Preservation of Identifiability): $(A_0,s_0) \in \mathbb{I}_{(x,n)}$ is identifiable with respect to some model $\mathbb{M}$ if and only if $(B_0,s_0) \in \mathbb{I}_{(y,w)}$ is identifiable with respect to $\mathbb{M}$.

Proof: ($\Rightarrow$) Let $(A_0,s_0) \in \mathbb{I}_{(x,n)}$ be identifiable with respect to $\mathbb{M}$. By Definition 3, there exists a single equivalence class in $\mathbb{I}_{(x,n)} \cap \mathbb{M}$. With Proposition 3, $(B_0,s_0) \in \mathbb{I}_{(y,w)}$ is identifiable with respect to $\mathbb{M}$.

($\Leftarrow$) Let $(B_0,s_0) \in \mathbb{I}_{(y,w)}$ be identifiable. Then all elements in $\mathbb{I}_{(y,w)} \cap \mathbb{M}$ are equivalent, i.e., for any $(A,s) \in \mathbb{I}_{(x,n)} \cap \mathbb{M}$, we have

$$f((A,s)) = (TA,s) \sim (B_0,s_0)$$

where T is the orthogonalization matrix given by (5). Hence there exists a permutation matrix P and a nonsingular diagonal matrix Λ such that

$$TA = B_0 P\Lambda$$
$$= TA_0 P\Lambda.$$

From (5), we have

$$U_s U_s^t A = U_s U_s^t A_0 P\Lambda. \tag{9}$$

Observe that $(A,s) \in \mathbb{I}_{(x,n)}$ implies $As(\cdot) = A_0 s_0(\cdot)$. Hence both A and A_0 have the same image space spanned by the column vectors of U_s. We then have

$$U_s U_s^t A = A$$
$$U_s U_s^t A_0 = A_0.$$

Substituting the above into (9), we have

$$A = A_0 P\Lambda.$$

Hence $(A,s) \sim (A_0,s_0)$. Therefore, $(A_0,s_0) \in \mathbb{I}_{(x,n)}$ is identifiable with respect to $\mathbb{M}$. $\square$

If $(B_0,s_0) \in \mathbb{I}_{(y,w)}$ is identified, the following corollary shows how to recover (A_0,s_0) in $\mathbb{I}_{(x,n)}$.

Corollary 1 (Recover A_0 from the Orthogonalized Parameter Matrix): If $(B,s) \sim (B_0,s_0)$, then $(T^\dagger B,s) \sim (A_0,s_0)$. Here $T^\dagger$ is the pseudo-inverse of T.

Proof: Let P and Λ be the permutation matrix and nonsingular diagonal matrix such that $s \sim s_0$, and

$$B = B_0 P\Lambda = TA_0 P\Lambda.$$

Consequently,

$$T^\dagger B = T^\dagger TA_0 P\Lambda$$
$$= U_s U_s^t A_0 P\Lambda = A_0 P\Lambda.$$

Hence $(T^\dagger B,s) \sim (A_0,s_0)$. $\square$

In summary, we have shown that there exists an orthogonalization transformation f that maps $\mathbb{I}_{(x,n)}$ to $\mathbb{I}_{(y,w)}$ in which the equivalence class containing (B^0,s^0) is an orthogonal equivalence class. Furthermore, f preserves all equivalence classes and, more importantly, identifiability. In addition, we have shown that A_0 can be "recovered" from $B \sim B_0$, the orthogonalized channel parameter matrix, even though f is not injective. Later in Section V, when we present the blind identification algorithms, we shall discuss how the orthogonalization transformation can be obtained from the statistics of the output observation $x(\cdot)$.

IV. Identifiable Models

In this section, R_{s_0} is further assumed to be diagonal, and we shall prove the identifiability results for two practical source models. In particular, we are interested in a model that the source signals are statistically uncorrelated or independent.

Theorem 2 (Identifiability of Uncorrelated Sources): (A_0,s_0) is identifiable with respect to $\mathbb{M}_1$, where $\mathbb{M}_1$ is the set of (A,s) satisfying the following condition:

(A1) $\{s_i(\cdot) i = 1,2,\cdots,m\}$ are uncorrelated

(A2) there is a $\tau > 0$ such that

$$\frac{E(s_i(t)s_i(t-\tau))}{E(s_i^2)} \neq \frac{E(s_j(t)s_j(t-\tau))}{E(s_j^2)}, \quad \text{for } i \neq j.$$

Proof: It needs to be shown that for any $(A,s) \in \mathbb{I}_{(x,n)} \cap \mathbb{M}_1$, $(A,s) \sim (A_0,s_0)$. From the definition of $\mathbb{I}_{(x,n)}$, for any $(A,s) \in \mathbb{I}_{(x,n)} \cap \mathbb{M}_1$, we have

$$As(\cdot) = A_0 s_0(\cdot).$$

Denote $R_s \equiv E\{s(t)s^t(t)\}$ and $R_{s_0} \equiv E\{s_0(t)s_0^t(t)\}$, we have

$$AR_sA^t = A_0 R_{s_0} A_0^t \tag{10}$$

where both R_s and R_{s_0} are diagonal because the source signals are uncorrelated as in (A1). Denote

$$R \equiv A_0 R_{s_0} A_0^t \tag{11}$$

and let R have a singular value decomposition of the following form:

$$R = U\Sigma U^t \tag{12}$$

where U is an $n \times m$ matrix with orthonormal column vectors and Σ is a positive definite diagonal matrix. With (10), (11), and (12), we have

$$U\Sigma U^t = A_0 R_{s_0} A_0^t = AR_sA^t. \tag{13}$$

Now define

$$T = \Sigma^{1/2} U^t. \tag{14}$$

Multiplying T on the left and T^t on the right, one obtains

$$TA_0R_{s_0}A_0^tT^t = I$$
$$TAR_sA^tT^t = I.$$

This implies that

$$V_0 \equiv TA_0R_{s_0}^{1/2} \quad (15a)$$
$$V \equiv TAR_s^{1/2} \quad (15b)$$

are orthogonal matrices. Since both R_s and R_{s_0} are diagonal, both TA_0 and TA belong to some orthogonal equivalence classes and T induces an orthogonalization transformation as defined in (8). Because the orthogonalization transformation preserves the identifiability as shown in Theorem 1, what is left to be shown is that $TA_0 \sim TA$, or equivalently, $V_0 \sim V$. Now let τ be such that the condition (A2) is satisfied. Similar to (10), we have

$$A_0R_{s_0}(\tau)A_0^t = AR_s(\tau)A^t$$

where $R_{s_0}(\tau) \equiv E\{s_0(t)s_0^t(t-\tau)\}$ and $R_s(\tau) \equiv E\{s(t)s^t(t-\tau)\}$ both are diagonal matrices. Consequently, we have

$$V_0R_{s_0}^{-1}R_{s_0}(\tau)V_0^t = VR_s^{-1}R_s(\tau)V^t. \quad (16)$$

Observe that the column vectors of the orthogonal matrices V and V_0 are eigenvectors of the matrix

$$R_\tau \equiv V_0R_{s_0}^{-1}R_{s_0}(\tau)V_0^t \quad (17)$$

while the diagonal entries of both $R_{s_0}^{-1}R_{s_0}(\tau)$ and $R_s^{-1}R_s(\tau)$ are eigenvalues of R_τ. Since the eigenvalues of R_τ are all distinct as assumed in (A2), there exists a permutation matrix P and a diagonal matrix J (with either 1 or -1 as its diagonal entries) such that

$$V = V_0PJ$$

or

$$V_0 \sim V.$$

Consequently, $TA_0 \sim TA$, and from Theorem 1, $A_0 \sim A$. □

Remark: The identifiable model in the above theorem is one that all the source signals $\{s_i(\cdot)\}$ are uncorrelated (as random processes) and with different autocorrelation at some $\tau > 0$.

Theorem 3 (Identifiability of Independent Sources): (A_0, s_0) is identifiable with respect to $\mathbb{M}_2$, where $\mathbb{M}_2$ is the set of (A, s) satisfying the following conditions:

(B1)

$\{s_i(t)i = 1,2,\cdots,m\}$ are mutually independent, and

(B2) $$\frac{E(s_i^4)}{E(s_i^2)^2} \neq \frac{E(s_j^4)}{E(s_j^2)^2}, \quad \text{for all } i \neq j.$$

Proof: For any $(A,s) \in \mathbb{I}_{(x,n)} \cap \mathbb{M}_2$, we have

$$As(\cdot) = A_0s_0(\cdot). \quad (18)$$

As in the proof of Theorem 2, the matrix T defined in (14) orthogonalizes the column vectors of A and A_0. Apply T to (18), we have

$$VR_s^{-1/2}s(\cdot) = V_0R_{s_0}^{-1/2}s_0(\cdot) \quad (19)$$

where V and V_0, as defined in (15), are orthogonal. Consequently,

$$E\left\{\left\|VR_s^{-1/2}s(\cdot)\right\|^2VR_s^{-1/2}s(\cdot)s^t(\cdot)R_s^{-1/2}V^t\right\}$$
$$= E\left\{\left\|V_0R_{s_0}^{-1/2}s_0(\cdot)\right\|^2V_0R_{s_0}^{-1/2}s_0(\cdot)s_0^t(\cdot)R_{s_0}^{-1/2}V_0^t\right\}.$$

One then obtains

$$VR_s^{-1/2}E\{ss^tR_s^{-1}ss^t\}R_s^{-1/2}V^t$$
$$= V_0R_{s_0}^{-1/2}E\{s_0s_0^tR_{s_0}^{-1}s_0s_0^t\}R_{s_0}^{-1/2}V_0^t. \quad (20)$$

Since $s(\cdot)$ and $s_0(\cdot)$ satisfy (B1), both $E\{s_0s_0^tR_{s_0}^{-1}s_0s_0^t\}$ and $E\{ss^tR_s^{-1}ss^t\}$ are diagonal. Denote

$$R_z = V_0R_{s_0}^{-1/2}E\{s_0s_0^tR_{s_0}^{-1}s_0s_0^t\}R_{s_0}^{-1/2}V_0^t$$
$$= VR_s^{-1/2}E\{ss^tR_s^{-1}ss^t\}R_s^{-1/2}V^t. \quad (21)$$

The column vectors of both V and V_0 are eigenvectors of R_z while the diagonal entries of both

$$R_{s_0}^{-1/2}E\{s_0s_0^tR_{s_0}^{-1}s_0s_0^t\}R_{s_0}^{-1/2}$$

and

$$R_s^{-1/2}E\{ss^tR_s^{-1}ss^t\}R_s^{-1/2}$$

are eigenvalues of R_z. In addition, with condition (B2), the eigenvalues of R_z are all distinct. As in the proof of Theorem 2, we have $V \sim V_0$, hence $A \sim A_0$. □

Remark: The identifiable model in this theorem is the one in which the source signals $\{s_i(t)\}$ are independent (as random variables) and with distinct kurtosis. Note the difference from the previous identifiable model.

In summary, we have presented two scenarios in which the actual channel parameter matrix and the source signals can be identified from the observation alone. Note that the conditions given in Theorems 2 and 3 are different conditions on the source signals, and one condition does not imply the other.

V. Blind Identification Algorithms

We now present blind identification algorithms for the two identifiable cases discussed in the previous section. In fact, the proofs of Theorems 2 and 3 suggest identification algorithms that identify the source signals via certain statistics of the observations. Indeed, the identification can be achieved by first orthogonalizing the channel parameter matrix. The orthogonalized parameter matrix can then be obtained from eigendecomposition.

5.1. Blind Identification of Independent Sources

We first present an algorithm for the blind identification of independent sources as described in Theorem 3. The algorithm, referred to as extended fourth-order blind identification (EFOBI), is an extension of the FOBI algorithm [16] which ignores the noise effect. As in the proof of Theorem 3, an orthogonalization transformation is first constructed. The orthogonalized channel parameter matrix is then obtained from a singular value decomposition of a fourth-order moment of the observation.

It is assumed in the following presentation that the noise has a covariance matrix of the following form:

$$R_n \equiv E\{n(t)n'(t)\} = \sigma^2 I$$

where σ^2 is unknown. For the case of accessible noise with general noise covariance structure, see [18].

EFOBI Algorithm

1) Estimate the output covariance $R_x \equiv E\{x(t)x'(t)\}$.
2) Compute an SVD of R_x:
$$R_x = [u_1, \cdots, u_n]\,\mathrm{diag}(\lambda_1^2, \lambda_2^2, \cdots, \lambda_n^2)[u_1, \cdots, u_n]'.$$
3) Estimate the number of sources m from the number of significant singular values, estimate the noise variance σ^2 from the insignificant singular values. (See proof for details.)
4) Perform an orthogonalization transformation. Let
$$d_i \equiv \sqrt{\lambda_i^2 - \sigma^2}, \qquad i = 1, 2, \cdots, m$$
$$U_s \equiv [u_1, \cdots, u_m]$$
$$T \equiv \mathrm{diag}\left(\frac{1}{d_1}, \frac{1}{d_2}, \cdots, \frac{1}{d_2}\right)U_s'$$
$$y(t) \equiv Tx(t).$$
5) Estimate the fourth-order moment:
$$M = E\{y(t)y'(t)y(t)y'(t)\}$$
and compute $\Delta M_n = \mathrm{diag}(\delta_1, \delta_2, \cdots, \delta_m)$,
$$\text{where } \delta_i = \frac{(m+4)\sigma^2}{d_i^2} + \frac{\sigma^2}{d_i^2}\left(\sum_{k=1}^{m} \frac{d_i^2 + \sigma^2}{d_k^2} + \frac{2\sigma^2}{d_i^2}\right).$$
6) Compute a singular value decomposition of $M - \Delta M_n$.
$$M - \Delta M_n = V \Sigma V'.$$
7) Channel estimation A_0: $\hat{A} = T^{\dagger} V$.
8) Signal estimation $s_0(\cdot)$: $\hat{s}(t) = V' y(t)$.
9) Stop.

Proofs and Remarks of EFOBI

Proposition 4 (Steps 1–4: Orthogonalization Transformation): The column vectors of TA_0 are orthogonal.

Proof: From (1), we have

$$R_x = A_0 R_{s_0} A_0' + \sigma^2 I. \tag{22}$$

The singular value decomposition of R_x must then have the following form:

$$R_x = [u_1, \cdots, u_n] \cdot \mathrm{diag}(d_1^2 + \sigma^2, d_2^2 + \sigma^2, \cdots, d_m^2 + \sigma^2, \sigma^2, \cdots, \sigma^2) \cdot [u_1, \cdots, u_n]'. \tag{23}$$

Therefore, one can obtain an estimation of the number of sources m, the noise variance σ^2 and then d_i's. See [12] for one of the many estimation methods. Comparing (22) and (23), one easily obtains

$$A_0 R_{s_0} A_0' = [u_1, \cdots, u_n]\,\mathrm{diag}(d_1^2, d_2^2, \cdots, d_m^2, 0, \cdots, 0) \cdot [u_1, \cdots, u_n]' = [u_1, \cdots, u_m]\,\mathrm{diag}(d_1^2, d_2^2, \cdots, d_m^2) \cdot [u_1, \cdots, u_m]'.$$

With T defined in Step 4, we have

$$TA_0 R_{s_0} A_0' T' = I. \tag{24}$$

Therefore, $TA_0 R_{s_0}^{1/2}$ is orthogonal. Since R_{s_0} is diagonal for independent sources, the column vectors of TA_0 are orthogonal. □

With $y(t)$ defined in Step (4), let

$$B_0 = TA_0 \tag{25a}$$
$$w(t) = Tn(t). \tag{25b}$$

We then have

$$y(t) = B_0 s_0(t) + w(t) \tag{26}$$

where the columns of B_0 are orthogonal. In fact, $B_0 R_{s_0}^{1/2}$ is an orthogonal matrix as shown in (24).

Proposition 5 (Steps 4 and 5):

$$M - \Delta M_n = B_0 R_{s_0}^{1/2}\,\mathrm{diag}(\kappa_1, \kappa_2, \cdots, \kappa_m) R_{s_0}^{1/2} B_0'$$

where

$$\kappa_i = \frac{E\{s_{0i}^4\}}{E\{s_{0i}^2\}^2}, \qquad i = 1, 2, \cdots, m.$$

Proof: For notational convenience, we use y, s, and w in the places of $y(t)$, $s(t)$, and $w(t)$, respectively. With M defined in Step 5 and (26), we have

$$M = B_0 E\{s_0 s_0' B_0' B_0 s_0 s_0'\} B_0' + B_0 E\{\|w\|^2 s_0 s_0'\} B_0' + E\{ww' s_0' B_0' B_0 s_0\} + 2E\{Bs_0 s_0' B_0' ww'\} + 2E\{ww' B_0 s_0 s_0' B_0'\} + E\{\|w\|^2 ww'\}.$$

With (24) and (25a), we have $B_0' B_0 = R_{s_0}^{-1}$ and

$$B_0 E\{s_0 s_0'\} B_0' = I.$$

Hence

$$M = BE\{s_0 s_0' R_{s_0}^{-1} s_0 s_0'\} B_0' + E\{\|w\|^2\} I + R_w E\{s_0' R_{s_0}^{-1} s_0\} + 4R_w + E\{\|w\|^2 ww'\}$$

where $R_w = E\{ww'\}$. Note that

$$E(s_0' R_{s_0}^{-1} s_0) = \mathrm{tr}\left(E\{R_{s_0}^{-1} s_0 s_0'\}\right) = m.$$

Hence, if we let

$$\Delta M_n \equiv (m+4) R_w + E\{\|w\|^2 ww'\} + E\{\|w\|^2\} I \tag{27}$$

we have

$$M - \Delta M_n = B_0 E\{s_0 s_0^t R_{s_0}^{-1} s_0 s_0^t\} B_0^t.$$

Since the components of s_0 are independent,

$$\begin{aligned} M - \Delta M_n &= B_0 \operatorname{diag}\left(E\{s_{0_1}^2\}\kappa_1, E\{s_{0_2}^2\}\kappa_2, \cdots, E\{s_{0_m}^2\}\kappa_m\right) B_0^t \\ &= B_0 R_{s_0}^{1/2} \operatorname{diag}(\kappa_1, \kappa_2, \cdots, \kappa_m) R_{s_0}^{1/2} B_0^t \end{aligned}$$

which is the equation given in Proposition 5.

We now show that ΔM_n defined by (27) is what is given in Step 5. The three items in (27) can be evaluated as follows.

$$\begin{aligned} R_w = E(ww^t) &= TR_n T^t \\ &= \sigma^2 TT^t = \sigma^2 \operatorname{diag}\left(\frac{1}{d_1^2}, \frac{1}{d_2^2}, \cdots, \frac{1}{d_m^2}\right) \end{aligned} \quad (28)$$

$$E(\|w\|^2) = \operatorname{tr}(E(ww^t)) = \sigma^2 \sum_{i=1}^{m} \frac{1}{d_i^2}. \quad (29)$$

For the term $E(\|w\|^2 ww^t)$, consider its (i,j)th entry $E(\|w\|^2 w_i w_j)$,

$$E\left(\|w\|^2 w_i w_j\right) = E\left(\sum_{k=1}^{m} w_\kappa^2 w_i w_j\right)$$

where w_i is the ith component of w. Observe that w has Gaussian distribution and $E(ww^t)$ is a diagonal matrix, hence $E(w_i w_j) = 0$ for $i \neq j$. This implies that w_i and w_j are independent whenever $i \neq j$. Therefore,

$$E\left(\|w\|^2 w_i w_j\right) = \begin{cases} 0, & \text{if } i \neq j. \\ E(w_i^4) + \sum_{k=1;\, k \neq i}^{m} E(w_k^2) E(w_i^2), & \text{otherwise.} \end{cases} \quad (30)$$

Recall that for the zero-mean Gaussian random variable, $E(w_i^4) = 3(E(w_i^2))^2$. Substituting (28), (29), and (30) into (27), one obtains ΔM_n as given in Step 5. □

Proposition 6 (Steps 6 and 7): There exists a permutation matrix P and a nonsingular diagonal matrix Λ such that $\hat{A} = A_0 P \Lambda$.

Proof: From Step 6 and Proposition 5, we have

$$\begin{aligned} M - \Delta M_n &= V \Sigma V^t \\ &= B_0 R_{s_0}^{1/2} \operatorname{diag}(\kappa_1, \kappa_2, \cdots, \kappa_m) R_{s_0}^{1/2} B_0^t. \end{aligned}$$

Since $B_0 R_{s_0}^{1/2}$ is orthogonal and the eigenvalues of $M - \Delta M_n$ are all distinct, $B_0 R_{s_0}^{1/2}$ and the orthogonal matrix V from an eigendecomposition are related by

$$B_0 R_{s_0}^{1/2} = VPJ \quad (31)$$

where P is a permutation matrix and J is a diagonal matrix whose diagonal entries are either $+1$ or -1. Recall that $B_0 = TA_0$, we have

$$\begin{aligned} T^\dagger VPJ &= T^\dagger T A_0 R_{s_0}^{1/2} \\ &= U_s U_s^t A_0 R_{s_0}^{1/2}. \end{aligned}$$

Since U_s and A_0 have the same image space, the above equation leads to

$$\hat{A} = A_0 R_{s_0}^{1/2} J P^t.$$

The proposition is proved because $R_{s_0}^{1/2}$ and J are diagonal and P is a permutation matrix. □

Remark on Step 8: From Step 8 and (26), we have

$$\begin{aligned} \hat{s}(t) &= V^t y(t) \\ &= V^t B_0 s_0(t) + V^t w(t). \end{aligned}$$

From (31), we have

$$\hat{s}(t) = PJR_{s_0}^{-1/2} s_0(t) + V^t w(t).$$

Hence $(\hat{A}, \hat{s}) \sim (A_0, s_0)$ when there is no noise. When noise is present, on the other hand, $\hat{s}(t)$ is a least-square estimate of $PJR_{s_0}^{-1/2} s_0(t)$, which is equivalent to $s_0(t)$.

5.2. *Algorithm for Multiple Unknown Signals Extraction (AMUSE)*

The success of EFOBI is due to the assumptions that the source signals are independent and their kurtosis are distinct. These assumptions have some shortcomings. For instance, EFOBI can not handle Gaussian sources because the fourth-order moments of Gaussian signals are completely specified from the second-order moments. Also, the estimates of higher order moments in practical situation usually exhibits larger variance. This will in turn affect the estimation of matrix A_0 and source vector s_0. To circumvent these shortcomings, AMUSE is developed. In the development of EFOBI, only the statistics of marginal distribution of the stochastic process are exploited. In contrast, AMUSE exploits the second-order statistics of the process $s_0(t)$.

The idea is in fact very simple. With the orthogonalized parameter matrix B_0 that satisfies (26), we have, for $\tau \neq 0$,

$$\begin{aligned} R_y(\tau) &= B_0 R_{s_0}(\tau) B_0^t \\ &= B_0 R_{s_0}^{1/2} R_{s_0}^{-1/2} R_{s_0}(\tau) R_{s_0}^{-1/2} R_{s_0}^{1/2} B_0^t \end{aligned}$$

where $R_y(\tau) \equiv E\{y(t) y^t(t-\tau)\}$ and

$$R_{s_0}(\tau) \equiv E\{s_0(t) s_0^t(t-\tau)\}$$

Note that $R_{s_0}(\tau)$ is diagonal and that the diagonal entries of $R_{s_0}^{-1/2} R_{s_0}(\tau) R_{s_0}^{-1/2}$ are all distinct for some τ as assumed in the Condition (A2) of Theorem 2. Let $R_y(\tau)$ have a eigendecomposition (Schur decomposition) of the following form:

$$R_y(\tau) = V \Sigma V^t.$$

We then conclude, as in the case of EFOBI, that

$$B_0 R_{s_0}^{1/2} = VPJ$$

where P is some permutation matrix and J is a diagonal matrix whose diagonal entries are either $+1$ or -1. We now summarize the AMUSE algorithm, which in most parts, is similar to the EFOBI algorithm.

The AMUSE Algorithm

1) Perform the orthogonalization transformation as in Steps 1–4 of the EFOBI Algorithm.
2) Select a τ such that $(\boldsymbol{R}_y(\tau)+\boldsymbol{R}_y(\tau)^t)/2$ has distinct eigenvalues, where $\boldsymbol{R}_y(\tau) \equiv E\{y(t)y(t-\tau)^t\}$.
3) Let $\boldsymbol{V}$ be the eigenmatrix obtained from the eigendecomposition of $(\boldsymbol{R}_y(\tau)+\boldsymbol{R}_y(\tau)^t)/2$.
4) Channel estimation $\boldsymbol{A}_0$: $\hat{\boldsymbol{A}} = \boldsymbol{T}^{\dagger}\boldsymbol{V}$.
5) Signal estimation $s_0(\cdot)$: $\hat{s}(t) = \boldsymbol{V}^t y(t)$.
6) Stop.

VI. Performance Evaluation and an Example

In this section, we evaluate the performance of FOBI, EFOBI, and AMUSE by some heuristic arguments and simulation results.

6.1. Comparison of FOBI and EFOBI Algorithms

In evaluating the FOBI and EFOBI algorithms, a parameter matrix is chosen (randomly) as

$$\boldsymbol{A}_0 = \begin{bmatrix} 0.9129 & 0.2491 \\ 0.1826 & 0.8305 \\ 0.3651 & 0.4983 \end{bmatrix}.$$

Two i.i.d. random sources are used. One i.i.d. sequence is drawn from a uniform distribution while the other i.i.d. random sequence is drawn from a normal distribution. Both sources are zero-mean and have unit variances. A vector Gaussian noise process is added to the observation with noise covariance $\sigma^2\boldsymbol{I}$. The signal-to-noise ratio (SNR) is then defined as

$$\mathrm{SNR} = 10\log_{10}\frac{1}{\sigma^2}\ (\mathrm{dB}).$$

The parameter matrix $\boldsymbol{A}_0$ is estimated by using EFOBI algorithm and FOBI algorithm. The column vectors of the estimated parameter matrix $\hat{\boldsymbol{A}}$ are normalized and arranged so that a comparison between $\boldsymbol{A}_0$ and $\hat{\boldsymbol{A}}$ can be made (remove the indeterminacy introduced by permutation and scaling). For a Monte Carlo simulation of N trials, the normalized root-mean-square error (NRMSE) is defined as

$$\mathrm{NRMSE} = 20\log\left(\frac{1}{\|A^0\|_F}\sqrt{\frac{1}{N}\sum_{k=1}^{N}\|\boldsymbol{A}_0 - \hat{\boldsymbol{A}}^{(k)}\|_F^2}\right)$$

where $\hat{\boldsymbol{A}}^{(k)}$ is the estimate of $\boldsymbol{A}_0$ at the kth Monte Carlo trial. Here, $\|\cdot\|_F$ is the Frobenius norm.

Fig. 3 shows the plots of NRMSE versus SNR along with the simulation conditions. The purpose of this simulation is to evaluate the performance of FOBI and EFOBI algorithms under the noise level from -10 dB to 20 dB. The simulation with data length of 2000 samples and 4000 samples are plotted. 100 Monte Carlo trials are conducted. As one can see, EFOBI does provide a better performance especially in the SNR range from 0 dB ~ 15 dB. The improvement is as much as around 5 dB. For high SNR, the performance of FOBI is of course close to that of EFOBI algorithm as shown in the plots at SNR level 20 dB.

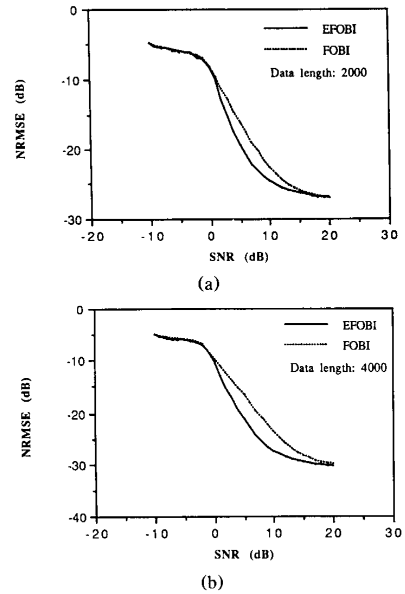

Fig. 3. (a) Comparison between FOBI and EFOBI with data length $Nl = 2000$. (b) Comparison between FOBI and EFOBI with data length $Nl = 4000$.

6.2. Comparison of EFOBI and AMUSE

In evaluating the performance of the AMUSE and EFOBI algorithms, the same parameter matrix is chosen as in the previous example. Two i.i.d. random sources are used. One i.i.d. sequence is drawn from a uniform distribution while the other one is drawn from a normal distribution. These two sequences are passed through two 20-tap FIR filters to facilitate the autocorrelation. The FIR filter used here is chosen to have an exponential decay impulse response of the following form:

$$h(n,\sigma) = \exp\left(\frac{-(n+1)}{10\sigma}\right), \qquad n = 0,1,2,\cdots,19.$$

The Gaussian i.i.d. sequence is passed through the FIR filter with $h(n,1)$ as its impulse response while the uniform i.i.d. sequence is passed through the FIR filter with $h(n,0.5)$ as its impulse response. The means of the two filtered sequences are then subtracted and variances unified. Again, a vector Gaussian noise process is added to the observation with noise covariance $\sigma^2\boldsymbol{I}$. The parameter matrix $\boldsymbol{A}_0$ is estimated by using AMUSE algorithm and EFOBI algorithm. The autocorrelation matrix used in the second Schur decomposition in AMUSE algorithm is chosen as $\boldsymbol{R}_y(5)$. The column vectors of the estimated parameter matrix $\hat{\boldsymbol{A}}$ are again normalized and arranged for comparison purposes.

Fig. 4 shows the plots of NRMSE versus SNR along with the simulation conditions. The simulations with data

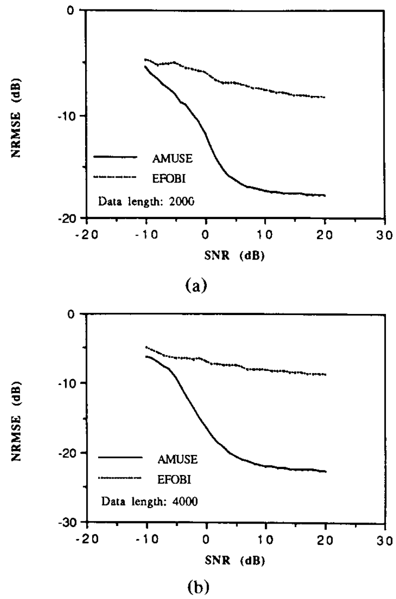

Fig. 4. (a) Comparison between AMUSE and EFOBI with data length = 2000. (b) Comparison between AMUSE and EFOBI with data length = 4000.

length of 2000 samples and 4000 samples are plotted. As one can see, the AMUSE algorithm performs significantly better than the EFOBI algorithm. Especially in the high SNR range where the noise is no longer a factor, the AMUSE algorithm provides around a 10-dB improvement. Since both algorithms provide accurate estimates when $Nl \to \infty$, where Nl is the sample size, this set of simulation shows that for a fixed Nl, the estimates of second order statistics (i.e., the autocorrelation matrices in AMUSE algorithm) are more accurate and reliable. Another interesting point is that EFOBI performs worse when the sources are not white. This can be seen by comparing the EFOBI curves in Figs. 3 and 4. Intuitively, this is due to the fact that when samples are correlated with each other, more samples are needed to obtain accurate estimates, especially in estimating higher order moments.

6.3. Separation of Speech Signals

Here we present an illustration that involves actual speech signals. Two IEEE standard speech signals sampled at 8000 Hz are mixed by $A_0 = \begin{bmatrix} 1.0 & 0.5 \\ 0.6 & 1.0 \\ 0.4 & 0.8 \end{bmatrix}$. The plots of the individual speech signals s_1 and s_2 are shown in Fig. 5(a) and (b). Speech s_1 is the sentence "Cats and dogs each hate the other" of a male speaker, while s_2 is the sentence "The pipe began to rust while new" of a female speaker. White Gaussian noise is added with equal energy level as the speech signals. The observed speech signals at three sensors are shown in Fig. 6(a), (b), and (c). Fig. 7(a) and (b) show speech signals estimated by AMUSE. It is clear that AMUSE works amazingly well in this case.

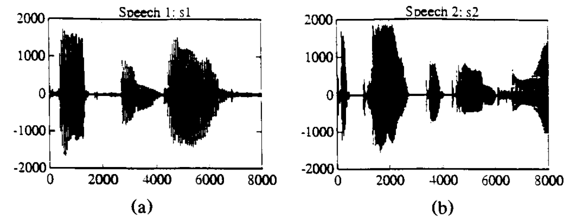

Fig. 5. (a) $s_1(\cdot)$. (b) $s_2(\cdot)$.

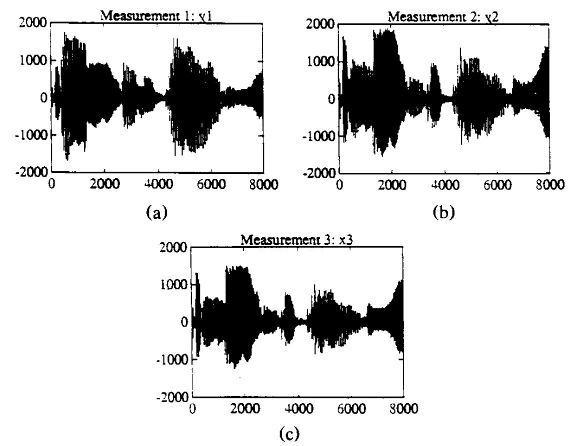

Fig. 6. (a) $x_1(\cdot)$. (b) $x_2(\cdot)$. (c) $x_3(\cdot)$.

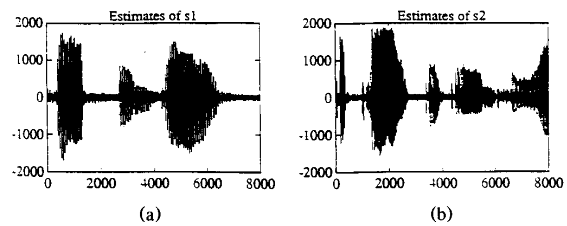

Fig. 7. (a) $\hat{s}_1(\cdot)$. (b) $\hat{s}_2(\cdot)$.

The purpose of this example is to test the algorithm when real speech signals are used. It may not reflect a real speech environment.

VII. Conclusion

In this paper, some fundamental issues of blind identification of source signals are considered, especially those of indeterminacy and identifiability. A mathematical structure of blind identification is developed. Based on this mathematical problem formulation, the issue of identifiability is investigated. Two blind identification algorithms that exploit different characteristics, namely, the fourth-order moment of marginal distribution and the second-order statistics of the random process, of the source signals are presented. Simulations have shown that EFOBI outperforms FOBI algorithm, and the AMUSE algorithm performs even better than EFOBI in case of non-white source signals. AMUSE is also applied to a speech extraction problem and shown to have promising results.

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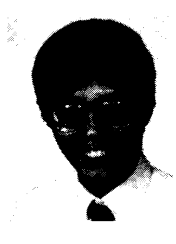

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