Blind Separation of Nonstationary Sources in Noisy Mixtures

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Abstract

In this letter we develop a new method of blind source separation that is not sensitive to additive white noise. Our method exploits the nonstationarity and temporal structure of sources. The method needs only multiple time-delayed correlation matrices of the observed data at several different time-windowed data frames to estimate the mixing matrix. The implementation using joint diagonalization (JD) is described. Simulations verify the high performance of the proposed method, especially in low SNR environment.

Indexing terms: Blind separation of noisy signals, joint diagonalization, noisy ICA, nonstationarity, pre-whitening for noisy data.

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1 Introduction

Blind source separation (BSS) is a fundamental problem which is encountered in a variety of signal processing problems where multiple sensors are involved. The task of BSS is to recover original sources from their linear instantaneous mixtures without resorting to any prior knowledge except for the statistical independence of sources. In the context of BSS, the n-dimensional observation vector $\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^T$ is assumed to be generated by

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is the unknown nonsingular mixing matrix, $\boldsymbol{s}(t)$ is the n-dimensional vector of sources that are assumed to be mutually independent and to be temporally correlated. In addition we assume that sources are nonstationary in the sense that their variances are time varying. The vector signal $\boldsymbol{v}(t)$ is additive white noise that is independent of $\boldsymbol{s}(t)$.

Methods of BSS rely on either learning the mixing matrix based on sample by sample or estimating the mixing matrix using statistics. The former approach is usually based on the gradient descent method and the latter is based on the eigen-decomposition. The eigen-decomposition methods exploit a set of higher-order cumulant matrices or a set of time-delayed correlation matrices. The mixing matrix is estimated by pre-whitening the data followed by a unitary transformation which jointly diagonalize cumulant matrices [2] or time-delayed correlation matrices [5, 4, 1].

The approach we take in this letter is based on the observation that the second-order correlation matrices of the data calculated at several different time-windowed data frames are sufficient for BSS when sources are nonstationary. We also employ a novel method of pre-whitening which diagonalize the time-delayed correlation matrix of the data so that the effect of white noise is negligible. The implementation using JD is described in next section. The comparison with some BSS algorithms is given in Section 3.

2 The New Method

Let us define a symmetric matrix $\boldsymbol{M}_x(t_k, \tau)$ by

$$\boldsymbol{M}_x(t_k, \tau) = \frac{1}{2}\left\{\boldsymbol{R}_x(t_k, \tau) + \boldsymbol{R}_x^T(t_k, \tau)\right\}, \tag{2}$$

where $\boldsymbol{R}_x(t_k, \tau) = E\{\boldsymbol{x}(t_k)\boldsymbol{x}^T(t_k - \tau)\}$ and $E\{\cdot\}$ denotes the statistical average operator. In practice, $\boldsymbol{R}_x(t_k, \tau)$ is evaluated using the samples in the kth time-windowed data frame. In similar manner, we define $\boldsymbol{M}_s(t_k, \tau)$ for source vector that is nonsingular diagonal matrix from the assumptions.

We observe that for any $\tau \neq 0$, the following relation holds:

$$\boldsymbol{M}_x(t_k, \tau) = \boldsymbol{A}\boldsymbol{M}_s(t_k, \tau)\boldsymbol{A}^T, \tag{3}$$

due the whiteness of noise. Note that for stationary sources, $\{\boldsymbol{M}_s(t_k, \tau)\}$ contain the same information regardless of t_k, whereas it is not true for nonstationary sources. In principle, two different matrices evaluated at t_1 and t_2 are sufficient for BSS, provided that $\boldsymbol{M}_s^{-1}(t_1, \tau)\boldsymbol{M}_s(t_2, \tau)$ has distinct diagonal elements. Under this condition, the mixing matrix $\boldsymbol{A}$ can be estimated using the generalized eigenvalue problem, i.e., the estimate of the mixing matrix $\widehat{\boldsymbol{A}}$ is obtained by solving

$$\boldsymbol{M}_x(t_1, \tau)\boldsymbol{M}_x^{-1}(t_2, \tau)\widehat{\boldsymbol{A}} = \widehat{\boldsymbol{A}}\boldsymbol{M}_s(t_1, \tau)\boldsymbol{M}_s^{-1}(t_2, \tau). \tag{4}$$

We refer to this method as nonstationary source separation using simultaneous diagonalization (NSS-SD). In practice, it is not clear which t_1 and t_2 guarantee the condition that $\boldsymbol{M}_s^{-1}(t_1, \tau)\boldsymbol{M}_s(t_2, \tau)$ has distinct diagonal elements. This drawback can be overcome by employing the joint diagonalization method [2]. This method will be referred to as nonstationary source separation using JD (NSS-JD).

Algorithm Outline: NSS-JD

(1) The pre-whitening is done using a time-delayed correlation matrix $\boldsymbol{M}_x(\tau)$ for some $\tau \neq 0$ in order to reduce the noise effect. The whitened data $\boldsymbol{z}(t)$ is $\boldsymbol{z}(t) = \boldsymbol{\Lambda}^{-\frac{1}{2}}\boldsymbol{U}^T\boldsymbol{x}(t)$ where $\boldsymbol{U}$ and $\boldsymbol{\Lambda}$ are the eigenvector and eigenvalue matrices of $\boldsymbol{M}_x(\tau)$, respectively.

(2) We divide the data into K non-overlapping blocks and calculate $\boldsymbol{M}_z(t_k, \tau)$ for $k = 1, \ldots, K$.

(3) We find a joint diagonalizer $\boldsymbol{V}$ of $\{\boldsymbol{M}_z(t_k, \tau)\}$ that satisfies

$$\boldsymbol{V}\boldsymbol{M}_z(t_k, \tau)\boldsymbol{V}^T = \boldsymbol{\Lambda}_k, \quad \text{for } k = 1, \ldots, K, \tag{5}$$

where $\{\boldsymbol{\Lambda}_k\}$ is a set of diagonal matrices. Then the estimate of the mixing matrix $\widehat{\boldsymbol{A}} = \boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{V}$.

Remark: At each data frame, t_k, we can calculate multiple time-delayed correlation matrices, $\{\boldsymbol{M}_z(t_k, \tau_j)\}$ for $j = 1, \ldots, J$ and find a joint diagonalizer of $\{\boldsymbol{M}_z(t_k, \tau_j)\}$ for $k = 1, \ldots, K$ and $j = 1, \ldots, J$. This method is referred to as NSS-TD-JD. For stationary sources, $\boldsymbol{M}_z(t_k, \tau_j) = \boldsymbol{M}_z(\tau_j)$. Then the method NSS-TD-JD becomes identical to the SOBI method [1] except for the different pre-whitening step.

3 Numerical Experiments

In this experiment, We used 3 digitized voice signals and 2 music signals, all of which were sampled at 8 kHz and generated five mixtures using randomly chosen mixing matrix. In order to evaluate the performance of algorithms, we calculated the performance index (PI) defined by

$$\mathrm{PI} = \frac{1}{2(n-1)} \sum_{i=1}^{n} \left\{ \left(\sum_{k=1}^{n} \frac{|g_{ik}|^2}{\max_j |g_{ij}|^2} - 1 \right) + \left(\sum_{k=1}^{n} \frac{|g_{ki}|^2}{\max_j |g_{ji}|^2} - 1 \right) \right\}, \tag{6}$$

where g_{ij} is the (i,j)-element of the matrix $\boldsymbol{G} = \widehat{\boldsymbol{A}}^{-1} \boldsymbol{A}$.

In NSS-SD, we computed two different time-delayed correlation matrices (with the time lag being 1 for both matrices) of the observation vector and employed the generalized eigen-decomposition to estimate the mixing matrix. In NSS-JD, we used 50 different non-overlapping data frames (200 data points in each frame) to calculate 50 different time-delayed correlation matrices (with the time lag being 1 for all of them) of the observation vector. Then, the joint diagonalization method was employed to estimate the mixing matrix. In NSS-TD-JD, we also used 50 different non-overlapping data frames. Using the data points in each frame we calculated 5 different time-delayed correlation matrices (say, $\tau = 1, \ldots, 5$). The the joint diagonalization method was employed to estimate the mixing matrix. The performance index of the methods are shown in Figure 1. At each SNR, we performed 10 different runs and averaged the value of performance index. Our methods NSS-TD-JD and NSS-JD showed the best performance in this experiment. The NSS-SD also successfully separated sources in low SNR (10dB-15dB), however, its performance was worse than both NSS-TD-JD and NSS-JD because only two different statistics was used. Other methods such as JADE [2], FLEXICA (one exemplary adaptive ICA algorithm) [3] and SOBI [1] showed good performance in high SNR.

4 Conclusion

We presented new methods of BSS when sources are nonstationary and temporally correlation in the presence of additive noise. The proposed methods required only multiple time-delayed correlation matrices to estimate the mixing matrix. The efficient implementation based on the joint diagonalization was explained. The robustness of the proposed methods (NSS-JD, NSS-TD-JD) in low SNR was verified by numerical experiments.

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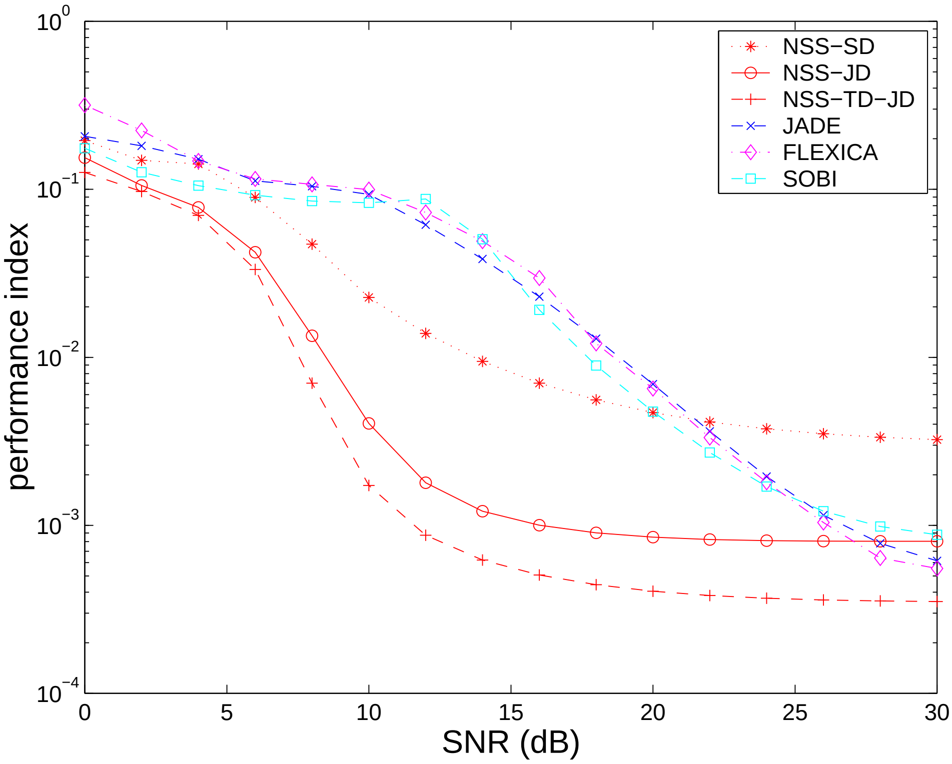

Figure 1: The comparison of performance of various BSS algorithms. Our methods (NSS-TD-JD, NSS-JD, NSS-SD) show improved performance, compared to JADE [2], SOBI [1], and FLEXICA [3], especially in low SNR.