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# ADAPTIVE APPROACH TO BLIND SOURCE SEPARATION WITH CANCELLATION OF ADDITIVE AND CONVOLUTIONAL NOISE

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## ABSTRACT

In this paper an adaptive approach to cancellation of additive, *convolutional* noise from many–source mixtures with a simultaneous blind source separation is proposed. Associated neural network learning algorithms are developed on the basis of decorrelation principle and energy minimization of output signals. The reference noise is transformed into a convolutional one by employing an adaptive FIR filter in each channel. Several models of NN learning processes are considered. In the basic approach the noisy signals are separated simultaneously with the additive noise cancellation. The simplified model employs separate learning steps for noise cancellation and source separation. Multi–layer neural networks improve the quality of results. Results of comparative tests of proposed methods are provided.

## 1. INTRODUCTION

Most approaches to blind source separation assume that sensor signals are noiseless or noise is considered as one of primary source [1] – [8]. More realistic and practical model is like this [9], [10]:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \qquad (1)$$

where $\mathbf{s}(t) = [s_1(t), ..., s_m(t)]^T$ is a vector of *unknown*, independent primary sources, $\mathbf{A}$ is a $m \times m$ unknown mixing matrix, $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^T$ is the *observed (measured)* vector of sensor signals and $\mathbf{n}(t) = [n_1(t), n_2(t)..., n_m(t)]^T$ is an additive noise vector. The noise signals $n_i(t)$ are independent of source signals $s_i(t)$, $\forall i$. Such kind of noise appears in almost all real–life (real–world) problems.

In the past we have developed efficient and robust learning algorithms for blind separation of 1–D signals and images [1], [5], [6]. We demonstrated by computer simulations the high performance and efficiency of proposed algorithms. These algorithms can extract all source signals even if some of them are extremely weak or the mixing matrix is very ill conditioned, assuming that additive noise $n_i(t)$ to each sensor is equal to zero. In fact we have assumed that noise signals is one of the unknown primary source signal which could be separated from other sources. Of course more than one source could be noise, but mostly one of them might be Gaussian noise.

The following problem arises: how efficiently to separate signals if additive noise could not longer be neglected ? Alternatively the problem can be stated as, how to cancel or reduce the additive noise. Thus our problem is how to modify existing algorithms to be still valid and efficient with additive noise vector. The main objective of this paper is to investigate this problem and to propose some constructive solutions.

In this paper we propose an adaptive approach to the cancellation of additive, *convolutional* noise from many–source mixtures that improves the performance of source separation. In the basic *demixing model* we simultaneously separate signals and subtract additive noise by employing adaptive FIR filter in each channel. We developed learning algorithms based on decorrelation principle and energy minimization of output signals. In a second model we attempt at first to reduce or to cancel noise and then to perform the blind separation of sources.

## 2. MODEL OF NOISE AND MIXTURE

In general, the problem is rather difficult because we have $2 \times m$ unknown signals (where $m$ is the number of sensors). Hence the problem is highly under-determined and without any a priori information about the mixture model and/or noise it is very difficult or even impossible to solve it [9], [10].

However, in many practical situation we can measure or model the environmental noise. Such noise we will denote further as *reference noise* $n_R(t)$ or a vector of reference noises (for each separate sensor $n_{Ri}(t)$) (Fig. 1). For example in acoustic *cocktail party* problem we could measure such noise during short salience period (when all persons do not speak) or we could measure and record such noise by an extra isolated microphone. In similar way we could measure noise in biomedical applications like EEG or EKG by extra electrodes.

This reference or environmental noise $n_R(t)$ influences each sensor but it could be added to mixture of signals with different strength. Moreover, noise could reach each sensor with some delay due to finite time propagation of signals. For this reason we assume in this work a model of additive and convolutional noise (called *Finite Impulse Response*, shortly the *FIR* model) [9], [11], i.e. (Fig. 1 (b)):

$$n_i(t) = \sum_{j=0}^{N_i-1} b_{ij} z^{-j} n_R(t) \qquad (2)$$

where $z^{-1} = e^{-sT}$ is unit delay. Such model is generally accepted as realistic (real–world) model in both areas of signal and image processing [9], [11]. Equation (2) could be

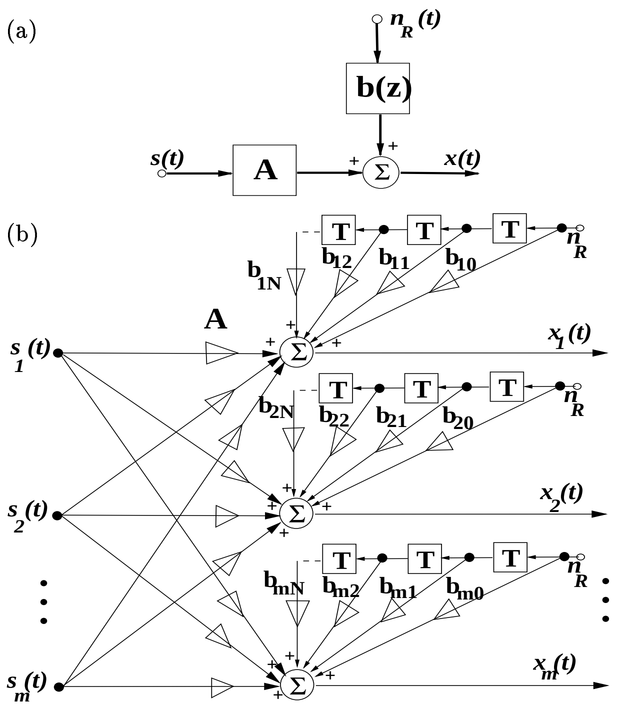



**Figure 1. The mixture and noise model: (a) source mixture with additive noise vector; (b) modeling the unknown noise by a convolutional additive noise ($n_R$ is the reference noise, the $b_{ij}$-s are unknown).**

written in time–domain as

$$n_i(t) = b_{i0}n_R(t) + b_{i1}n_R(t-T) + ... + b_{iN}n_R(t-NT). \quad (3)$$

In this model we assume that known reference noise is added to each sensor (mixture of sources) with different unit delays $T$ and various but unknown coefficients $b_{ij}(t)$. In other words we assume that noise is convoluted and reference noise $n_R$ is known, i.e. a priori information about noise is given:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{b}(z)n_R(t), \quad (4)$$

where $\mathbf{b}(z) = [b_1(z), b_2(z), ..., b_n(z)]^T$ with

$$b_i(z) = b_{i0} + b_{i1}z^{-1} + ... + b_{iN}z^{-N}. \quad (5)$$

Matrix $\mathbf{A}$, vector $\mathbf{b}(z)$ and the number of time delay units $N$ (i.e. maximum order of FIR filters) are completely unknown.

## 3. DEMIXING NEURAL NETWORK MODELS

In this section we describe the basic model for simultaneous source separation and noise cancellation as well as a simplified model.

### 3.1. Basic model

In the basic approach two learning steps are simultaneously performed: the signals are separated from their linear mixture and the additive noise is estimated and subtracted (Fig. 2). Thus the output signals are derived as:

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{W}\mathbf{x}(t) - \mathbf{h}(z)n_R = \\ &= \mathbf{W}\mathbf{A}\mathbf{s}(t) + \mathbf{W}\mathbf{b}(z)n_R - \mathbf{h}(z)n_R, \end{aligned} \quad (6)$$

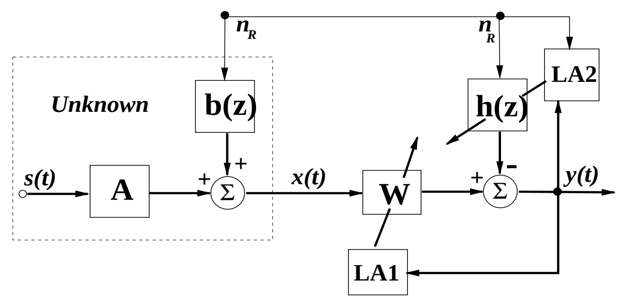



**Figure 2. The basic demixing model with two simultaneous learning steps ($LA1, LA2$ means learning algorithms).**

where $\mathbf{h}(z) = [h_1(z), ..., h_n(z)]^T$ with

$$h_i(z) = h_{i0} + h_{i1}z^{-1} + h_{i2}z^{-2} + ... + h_{iM}z^{-M} \quad (7)$$

A particular output signal is then:

$$y_i(t) = \tilde{y}_i(t) - n_i(t) = \tilde{y}_i(t) - \sum_{j=1}^{M} h_{ij}n_R(t-jT) \quad (8)$$

For simplicity of consideration let us assume that signals from $\mathbf{y}(t)$ are properly scaled and ordered in accordance with $\mathbf{s}(t)$. Then, it can be seen that $\mathbf{y}(t) \simeq \mathbf{s}(t)$ if

$$\mathbf{W}\mathbf{A} = \mathbf{I} \quad \text{and} \quad (9)$$

$$\mathbf{h}(z) = \mathbf{W}\mathbf{b}(z), \quad \text{i.e.} \quad h_i(z) = \sum_{j=1}^{m} w_{ij}b_j(z), \forall i. \quad (10)$$

The number of time delay units $M$ in the demixing model should be at least equal to the corresponding number $N$ in the mixing model (i.e. $M \geq N$).

### 3.2. Alternative simplified model

In a simplified model the two learning steps are performed in sequence. Now we attempt first to cancel noise contained in the mixture and then to separate the sources (Fig. 3). Thus the output signals are derived from:

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{W}[\mathbf{x}(t) - \mathbf{h}(z)n_R] = \\ &= \mathbf{W}\mathbf{A}\mathbf{s}(t) + \mathbf{W}\mathbf{b}(z)n_R - \mathbf{W}\mathbf{h}(z)n_R \end{aligned} \quad (11)$$

so it is obvious that $\mathbf{y}(t) \simeq \mathbf{s}(t)$ if (again, problems of signal scaling and permutation are omitted):

$$\mathbf{W}\mathbf{A} = \mathbf{I} \quad \text{and} \quad \mathbf{h}(z) = \mathbf{b}(z). \quad (12)$$

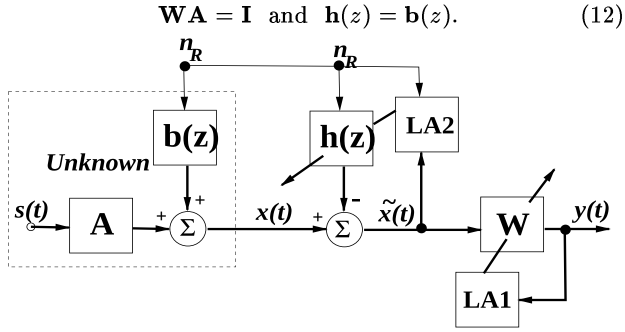



**Figure 3. Alternative (simplified) model for blind separation with noise cancellation.**

## 4. BASIC LEARNING ALGORITHMS

In this section we describe the adaptive learning rules, that perform simultaneous noise cancellation and blind source separation, according to specified demixing model.

### 4.1. Separation learning rules

The two learning algorithms $LA1$ and $LA2$ (Fig. 2, 3) are summarized by two following on–line rules. The synaptic weights $w_{ij}$ are updated according either to the

- global, robust rule (in single layer network) [1], [3],

$$\Delta \mathbf{W}(t+1) = \eta(t) \left\{ \mathbf{I} - \mathbf{f}[\mathbf{y}(t)]\mathbf{g}[\mathbf{y}^T(t)] \right\} \mathbf{W}(t), \qquad (13)$$

- or local (simplified) rule (in a multi–layer neural network) [5],

$$\Delta \mathbf{W}(t+1) = \eta(t) \left\{ \mathbf{I} - \mathbf{f}[\mathbf{y}(t)]\mathbf{g}[\mathbf{y}^T(t)] \right\}. \qquad (14)$$

### 4.2. Noise cancellation learning rules

The synaptic weights $h_{ij}$ are updated according to the general rule:

$$h_{ij}(t+1) = h_{ij}(t) + \tilde{\eta}(t) f_R[p_i(t)] n_R(t - jT), \quad (15)$$

where $p_i(t) = y_i(t)$ for the basic model in Fig. 2 (and in Fig. 4(a)), and $p_i(t) = \tilde{x}_i(t)$ for the simplified model in Fig. 3 (and in Fig. 4(b)).

$f_R()$ is a suitable chosen nonlinear function. Typically: $f_R(y_i) = y_i$, $f_R(y_i) = y_i^3$.

## 5. IMPROVED MULTI–LAYER MODELS

In order to improve the learning performance multi–layer neural networks could be used for both basic and simplified models (Fig. 4). These models perform separation and noise elimination by using multi–layer networks with pre– or post–processing noise cancellation steps.

At first, employing many layers is justified if we want to apply the local learning rule (14) for separation of mixtures in which some signals are very weak or the mixing matrix $\mathbf{A}$ is ill–conditioned. Secondly, a multi–layer model might be a proper solution if the initialization and decreasing speed of learning rates $\eta(t)$ and $\tilde{\eta}$ have not been chosen optimally. In order optimally to choose them the noise level contained in the mixtures should be known in advance, otherwise specific methods for the learning rate adaptation itself may be considered [4].

## 6. EXAMPLE OF SIMULATION RESULTS

In an illustrative example three (unknown) natural images were mixed by a randomly chosen matrix $\mathbf{A}$. Convolutional noises were modeled using two 5-th order FIR filters (N=5), the first one is introducing small noise and the second one – large noise (see example on Fig. 7). In the demixing model the number $M$ of delay units was chosen to be equal to 25. We used all proposed models for noise cancellation and blind separation of such generated sensor signals. Due to limit of space we compare the basic and simplified models only.

In order to estimate the separation quality we assume to know the original sources and the mixing matrix. Then the quality of obtained results is provided in two ways:

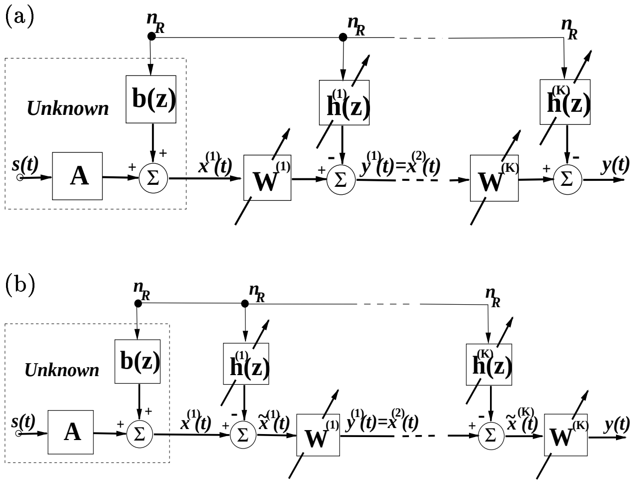



**Figure 4. Alternative multi–layer neural network models for blind separation with noise cancellation.**

1. individually for each source, by estimating a PSNR (peak signal to noise ratio) between the reconstructed source and the corresponding original source;
2. for the whole separated signal set, by calculation of a normalized error index $EI$, which is defined as:

$$EI = \sum_{j=1}^{m} \left( \sum_{i=1}^{n} \frac{|p_{ij}|^2}{max_i |p_{ij}|^2} - 1 \right). \qquad (16)$$

The $p_{ij}$-s are entries of a normalized matrix $\mathbf{P}(t) \in R^{n \times m}$ , $\mathbf{P} = \mathbf{W}(t)\mathbf{A}$ .

Performance results are summarized in Tab. 1. It should be emphasized that performance depends on learning rates $\eta(t), \tilde{\eta}(t)$ – on proper setting of their initial values and decay factors – and on chosen nonlinear activation functions $f, g, f_R$ . Provided result samples show that for given set of parameter values both models performed well and gave nearly similar results.

**Figure 5. Three original images (assumed to be completely unknown).**

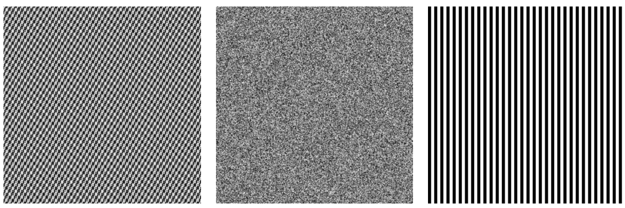
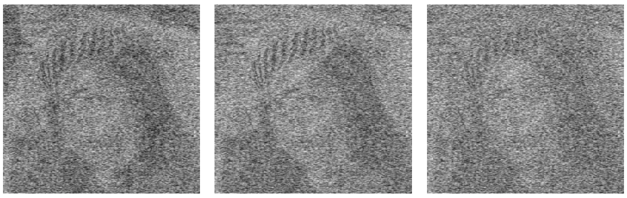


**Figure 6. Three alternative reference noises used in our computer tests (one noise per test) for generation of additive convolutional noise mixtures.**

Three sensor images (with small additive and convolutional noise 1)



Three sensor images (with large additive and convolutional noise 2)

After noise cancellation (in simplified model)

After complete blind separation and noise cancellation (in basic or simplified model)

**Figure 7. Example of blind source separation and noise cancellation of an image mixture with additive, convolutional noise.**

## 7. CONCLUSION

An adaptive approach for the restoration of unknown source signals from their signal mixtures distorted by additive noise was developed. The approach is valid under assumption that the unknown noise can be modeled as a convolutional noise mixture of a known reference noise.

The approach was tested on image sources, mostly natural face images, but is generally applicable to various classes of non–Gaussian signals, also to speech signals and biomedical signals. Preliminary computer experiments are very promising.

The proposed noise model could be extended to IIR adaptive filters, gamma filters and other more sophisticated models of noise. However, the open problem is how to cancel a non–additive noise and how to proceed if no reference noise or no knowledge about signal noise statistics are available. We hope to solve these problems in the future by using nonlinear neural filters.

| Noise | Signals | $EI$ | PSNR [dB] | | |
|---|---|---|---|---|---|
| | | | $Face$ 1 | $Face$ 2 | $Face$ 3 |
| Basic model | | | | | |
| small | $\mathbf{y}$ | 0.090 | 20.32 | 24.50 | 31.54 |
| large | $\mathbf{y}$ | 0.064 | 20.30 | 27.53 | 32.29 |
| Simplified model | | | | | |
| small | $\tilde{\mathbf{x}}$ | 1.427 | 10.42 | 13.82 | 12.91 |
| | $\mathbf{y}$ | 0.090 | 23.48 | 23.74 | 27.41 |
| large | $\tilde{\mathbf{x}}$ | 1.427 | 10.41 | 13.83 | 12.92 |
| | $\mathbf{y}$ | 0.077 | 22.96 | 24.89 | 28.58 |
| Separation of a noise–free mixture | | | | | |
| none | $\mathbf{y}$ | 0.079 | 23.37 | 26.19 | 28.36 |

**Table 1. Error index and quality factors for different models of separation and noise cancellation.**

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